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EDITORIAL

On the Use of Equivalent Circuits to Represent the Valve

IN this number we publish an interesting letter on this subject from an American reader. It raises some interesting points to which we wish to refer. As is now generally known, the valve can be represented either as a constant-voltage generator with a series resistance or as a constant-current generator with a shunt resistance. In his letter Dr. Salzberg discusses the relative merits of these two alternatives and comes to the conclusion that the former is a more fundamental concept, to which the latter has only a limited equivalence. We must confess that we had some doubts about the correctness of this conclusion, but on further consideration it appears that it has some foundation.

Before considering equivalent circuits let us look at the facts of the actual circuit consisting of a triode valve with a non-inductive resistance R in the anode circuit and a battery of voltage V_b and negligible resistance as the source of supply. Under steady conditions let the current be I_0 and the steady anode voltage V_a, then, in the absence of any signal voltage, there will be a steady power supply of $I_0^2R = I_0(V_b - V_a)$ to the load and of I_0V_a to the valve, the latter being dissipated in anode heating. On applying a signal voltage to the grid, an alternating current is superposed upon the steady current, and the power supplied to the load increases to $I_0^2R + \frac{\hat{\imath}^2}{2}R$, but it is important to note that the output of the battery has not increased, since the mean value of the current is unchanged. Where then does the load obtain its additional power? The answer to this is seen at once when the power supplied to the valve is calculated. Since any increase of current causes a decrease of anode voltage, the power is the mean value of $(I_0 + \hat{\imath}\sin\alpha)(V_a - \hat{v}\sin\alpha)$ which is $I_0V_a - \frac{\hat{\imath}\hat{v}}{2} = I_0V_a - \frac{\hat{\imath}^2}{2}R$. Hence the power supplied to the valve decreases by the same amount as that supplied to the load increases, and, although all the power comes from the battery if one takes the steady power distribution as a basis, one can say that, on applying a signal voltage to the grid, power is supplied to the load from the valve; that is to say, power, that would otherwise be dissipated in the valve, is transferred from the valve to the load.

From this point of view the valve may be regarded as the source of the signal-frequency power, acting as a convertor, converting some of its d.c. supply which, in the absence of a signal voltage, was all dissipated at the anode, into a.c. power and supplying it to the load.

The question then arises, how we should picture this fictional a.c. generator, and we have the two alternatives ; either an electromotive force μv_g with a series resistance ρ or a current $g_m v_g$ with a shunt resistance ρ. Use and wont would cause most people to favour the former, but that does not necessarily imply that it is any more fundamental than the latter. It may be objected that the latter entails two streams of electrons moving in opposite directions inside the valve but, in view of the fictional character of the whole procedure, this may perhaps be regarded as a minor detail. It is certainly not in its favour.

Both equivalent circuits give the same result for the power supplied to the load, but they give different results for the power dissipated in the internal resistance, that is to say, they agree on the measurable fact but differ in the case of the valve, on the unmeasurable fiction.

Fig. 1(a) shows the power actually supplied from the battery to the valve and load. Here $I^2 = i^2/2$ and I^2R is the amount by which the load power has increased and the valve power decreased due to the signal voltage on the grid. Fig. 1(b) shows the

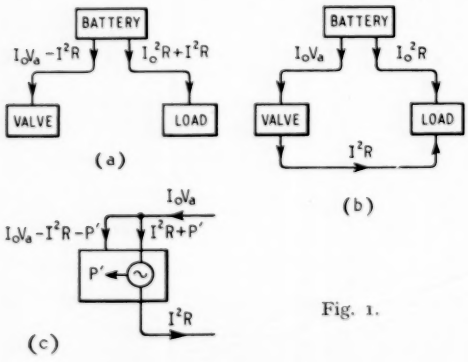

Fig. 1.

equivalent circuit fiction, according to which the supplies from the battery have remained unchanged, but the valve, functioning to some extent as a convertor, supplies the power I^2R to the load. Fig. 1(c) shows in more detail what happens, or is supposed to happen, in the valve. Of the steady battery supply I_0V_a, a portion is involved in the conversion ; if the convertor had 100 per cent efficiency this would be I^2R, but on account of losses the input to the convertor has to be $I^2R + P'$, the remainder $I_0V_a - I^2R - P'$ of the valve input taking no part in the conversion, but being dissipated in the valve. Of the $I^2R + P'$ supplied to the convertor, the part P' represents losses and is added to the dissipation, which is thus increased to $I_0V_a - I^2R$ as is evident in Fig. 1(c).

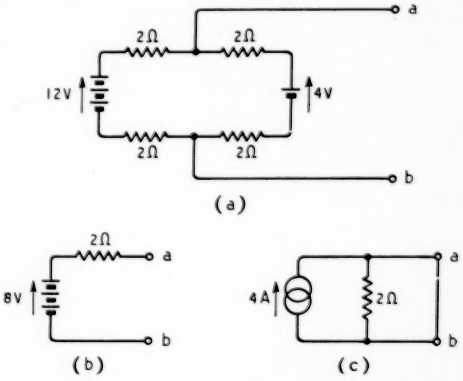

Fig. 2.

The losses P' cancel out, whatever their value may be, and do not exist as a separate entity ; they merely affect the fictitious division of the input in Fig. 1(c) and can be given any value by assuming a suitable efficiency of conversion.

Helmholtz's make-and-break theorem gives an e.m.f. and a series resistance which give the same current through an external load as the actual network, however complex it may be and however many sources of e.m.f. it may contain. Similarly the dual theorem gives a constant current and a shunt resistance which give the same external effects as the complex network. We do not think that it has ever been suggested that the internal losses are the same in the two cases, nor that one case is any more fundamental in conception than the other. In both cases the equivalence is only concerned with the external load.

To make this quite clear we may turn from complex fictions and consider the simple network shown in Fig. 2(a). The open-circuit voltage across *ab* is obviously

8 volts and the resistance of the passive network between a and b 2 ohms ; hence the series equivalent circuit is as shown in Fig. 2(b). If a and b in Fig. 2(a) are short-circuited the current is 4 amperes ; hence the parallel equivalent circuit is as shown in Fig. 2(c) in which the constant-current generator is shunted by a resistance of 2 ohms. If we now assume a load resistance of say 6 ohms connected between a and b, the power supplied to it is easily seen to be 6 watts in each case, but the internally dissipated power is 10 watts in the actual network, 2 watts in the series equivalent and 18 watts in the parallel equivalent circuit. Hence the circuits are only equivalent as regards the external load ; their internal losses are entirely different from those of the actual network. It should be noted, however, that in the absence of any external load there are losses in the network amounting to 8 watts on open circuit and to 40 watts on short circuit. On connecting the 6 ohm load the internal losses are increased from 8 to 10 watts, which agrees with the 2 watts internal loss in the series equivalent circuit, but not with the 18 watts in the parallel equivalent circuit.

According to the principle of duality, however, having in the series circuit assumed a load of 3 times the internal resistance, we should in the parallel circuit assume a load having 3 times the internal conductance ; i.e., 1.5 mhos or 2/3 ohm. The constant current of 4 amperes will divide proportionally to the two conductances, and the 3 amperes through the load of 1.5 mhos gives 6 watts as before while the 1 ampere through the 0.5 mho gives an internal loss of 2 watts as in the series equivalent circuit with the load of 6 ohms.

Circuit	Internal dissipation in watts			
	Open circuit	Short circuit	With 6-ohm load	With 2/3-ohm load
Actual ..	8	40	10	26
Series equivalent	0	32	2	18
Parallel equivalent	32	0	18	2

The Table shows the internal dissipation of the actual and equivalent circuits under the various conditions. The figures bring out clearly the principle of duality between the two equivalent circuits, but they also show equally clearly that the series equivalent is the only one that bears any simple relationship to the actual circuit. In each case the internal losses in the actual circuit exceed those in the series equivalent circuit by 8 watts, that is, by the losses on open circuit. Hence the series circuit gives the power supplied to the load and also the increase of internal loss due to the application of the load. The parallel circuit gives the power supplied to the load, but not the internal loss directly ; it can be obtained by modifying the load in a manner that is of little more than academic interest. We are pleased, therefore, to publish Dr. Salzberg's letter drawing attention to what he calls the limited nature of the equivalence.

G.W.O.H.

VISUAL MEASUREMENT OF RECEIVER NOISE*

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SUMMARY.—A rapid and frequently-used method of estimating the equivalent input noise power of a receiver is to inject a pulse-modulated carrier into the receiver input and to observe the output voltage on a cathode-ray oscilloscope; the input power is adjusted until a certain relation between the magnitudes of the output pulse and noise is observed. The equivalent signal-to-noise ratio at the receiver input then has some reference value.

Experiments are described in which the reference value of signal-to-noise ratio was measured. Specific instructions defining the relation to be observed between the output pulse and the noise were given to the operators. The precision of the settings obtained by operators is investigated. The application of the visual method to the determination of noise factor and the effect of including a video filter in the measuring equipment are discussed.

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1. Introduction
2. Description of Apparatus
3. Experiments
4. Experimental Results
5. Application of Results
6. Conclusions

Appendix

1. Introduction

THE wide use of pulse technique in recent years has suggested a rapid and simple method of measuring receiver noise. The method is visual; a square-wave modulated output from a signal generator with an appropriate output impedance is applied to the input of a receiver and the output is displayed on a cathode-ray oscilloscope. The method, called sometimes the "Army setting", consists of adjusting the signal-generator output so that the top of the noise in the spaces is at the same level as the bottom of the noise modulating the top of the pulse. When the signal-generator output is so adjusted, the i.f. output signal-to-noise ratio during the pulses has a value which is not very accurately repeatable owing to the subjective nature of the adjustment; it may be expected, however, that the value should not differ much from some reference value. When this reference value of i.f. output signal-to-noise ratio, which we shall write $(S/N)_0$, is known, the equivalent noise power at the receiver input may be determined from the signal-generator output power at setting.

A diagram of the appearance of the oscilloscope screen at setting is given in Fig. 1(a). The top of the detected pulse is modulated with noise ("modulating noise") and noise also appears in the spaces ("free noise"). The probability distributions of the free and modulating noise depend on the law of the detector and amplifiers. For example, if a linear envelope detector and linear amplifiers are used, the free-noise output has a Rayleigh distribution while the modulating noise, for reasonably high signal-to-noise ratios, has approximately a Gaussian distribution symmetrical about the pulse top. These distributions are shown in Fig. 1(b). It can be seen from the shape of the curves that the top of the free noise and the bottom of the modulating noise are not well defined and, consequently, the estimates of their levels will vary.

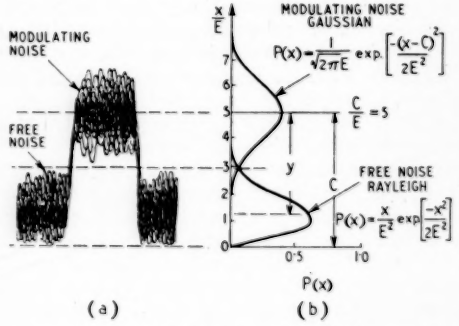

Fig. 1. (a) *Appearance of oscilloscope screen,* (b) *distributions of free and modulating noise.*

Experiments were carried out with the object of estimating the reference value of signal-to-noise ratio $(S/N)_0$. It was thought that operators who had already made measurements of this type might have

* MS. accepted by the Editor, July 1946.

acquired a bias and, to avoid this, inexperienced operators were used and definite verbal instructions were given to them. In this way it was intended to obtain results which could be reproduced independently by any other inexperienced operator following the same instructions. Apart from finding the reference value of signal-to-noise ratio $(S/N)_0$, the precision of the method and the variation of the means of operators readings were also investigated.

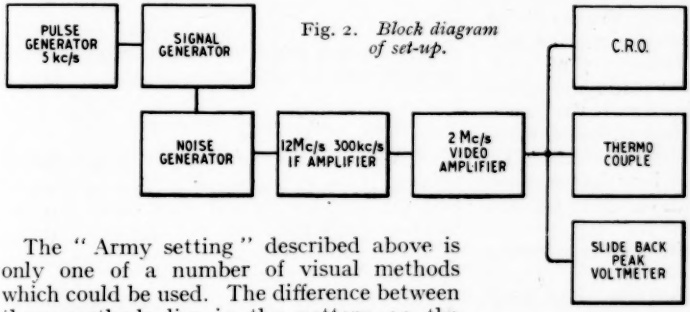

Fig. 2. *Block diagram of set-up.*

The "Army setting" described above is only one of a number of visual methods which could be used. The difference between these methods lies in the pattern on the oscilloscope screen taken to define the reference signal-to-noise ratio. An example of an alternative method is the so-called "2/1 setting," in which the height of the top of the modulating noise is adjusted to be twice that of the free noise.*

It can be seen that a point in favour of the Army setting is that it refers to one deflection level and so the reference value of signal-to-noise ratio $(S/N)_0$ should be independent of the detector law and of non-linearities in the amplifying system. This would not be the case with the 2/1 setting which refers to two different levels.

2. Description of Apparatus

In using the method it is necessary for the square-wave modulation to be complete; i.e., the carrier must have zero amplitude in the spaces. Also an appreciable part of the top of the detected pulse must be flat, and therefore, the square pulses of carrier applied to the receiver must have a length at least equal to several times the reciprocal of the i.f. bandwidth.

In the experiments the video-amplifier bandwidth was greater than the i.f. bandwidth. This ensured that little frequency distortion of the noise occurred after detection. If the video bandwidth of a receiver is narrower than the i.f. bandwidth, the output should be taken from the detector and amplified in a separate amplifier of adequate bandwidth.

A block diagram of the experimental set-up is shown in Fig. (2). The signal generator was modulated by square pulses from the 5-kc/s pulse generator. A diode noise source was used and the signal and noise were combined in the noise-generator chassis. The output of signal and noise was then applied to a 12-Mc/s, 300-kc/s bandwidth i.f. amplifier followed by a linear envelope detector. After amplification in a 2-Mc/s bandwidth video amplifier the detector output was applied to a Cossor oscilloscope with a J-type blue screen. The noise at the video output was measured in the absence of signal by a thermocouple. The signal was measured by determining the height of the output video pulse in the absence of noise with a slide-back peak voltmeter with an accuracy of about 1%. A hood was fitted to the c.r. tube. The time-base velocity was such that individual noise peaks could be resolved easily when the c.r. tube spot was well focused. The length of the pulse was about 30μsec so that several detected noise peaks occurred during a pulse.

3. Experiments

The readings obtained by a single operator using the visual method will be distributed about their mean value but the mean values of different operators will also vary. Part of this variation may be expected to arise from the use of different criteria in locating the top of the free noise and the bottom of the modulating noise. Two possible criteria would be (i) to estimate the level at which the mean brilliancy is a given fraction of the maximum brilliancy, (ii) to estimate the level which is crossed by noise peaks at a certain low rate. Both

* This method is mentioned in "Radio Measurements in the Decimetre and Centimetre Wavebands." R. J. Clayton, J. E. Houldin, H. R. L. Lamont, and W. E. Willshaw. *Journ. Instn. Elect. Engrs.*, 1946, Vol. 93, Part III, p. 97.

(i) and (ii) depend directly on the probability distribution while (ii) also depends on the i.f. bandwidth in so far as, for a constant r.m.s. output noise voltage, the number of noise peaks crossing any level is proportional to the i.f. bandwidth.

In an attempt to distinguish between these two criteria three variations (A,B,C) of the method were examined. Rather vague verbal instructions were given in A. In B the instructions were worded so as to direct the operator's attention to criterion (ii). By defocusing the c.r.-tube spot until individual noise peaks were not resolvable it could be arranged that only criterion (i) was operative; this was done in C. for which variation the same instructions were given as in A. Identical verbal instructions were given to each operator. The results should therefore show how the reference signal-to-noise ratio depends on the resolution of the noise peaks.

The instructions given in the three variations were as follows.

A & C. "Keep bottom of top noise level with top of bottom noise."

B. "Adjust input so that top noise is just completely above the bottom noise; i.e., so that no peaks overlap."

No additional explanation of the instructions was given.

Six operators were employed and each operator made a run of six settings for each variation. The experiments were carried out in the order A, B, C, and each variation was done separately with an interval of a few hours.

4. Experimental Results

Variation	A	B	C
$(S/N)_0$ db	8.3	11.2	8.8
σ_1 db	1.2	1.2	1.0
σ_2 db	0.5	0.3	0.5
σ_3 db	1.3	1.3	1.2

The reference signal to noise ratio $(S/N)_0$ is the mean of all readings of all operators in each variation.

σ_1 indicates the difference to be found amongst operators following the same instructions. It is the standard deviation of observers' means.

σ_2 indicates the precision of an operator. It is the mean of the standard deviations of each operator.

σ_3 indicates the error of a single measurement. It is the standard deviation of all readings in each section.

All averaging was carried out in voltage readings and the results were converted into decibels as given in the Table. From the readings made at the video output the corresponding i.f. signal-to-noise ratios were calculated. If C is the pulse amplitude at the video output then the i.f. signal-to-noise ratio (S/N) is given by

$$(S/N) = \frac{C}{\sqrt{2}} \cdot \frac{1}{E},$$

E is obtained from the video output r.m.s. noise voltage by dividing by

$$\sqrt{2 - \frac{\pi}{2}} = 0.655.$$

E would be the i.f. output r.m.s. noise voltage if no video amplification were used and the detector output were exactly equal to the amplitude of the input voltage. Known small detector corrections were applied in calculating C and E.

The results show a statistically significant agreement between the values of $(S/N)_0$ obtained for A and C. This shows that the ability to resolve individual noise peaks is not important when vague instructions as in A are given. In fact, it may be that with these instructions the brilliancy criterion is dominant even when the resolution of noise peaks is possible. In any case, when following the instructions of A, the focusing of the c.r. tube would not be critical. It may be assumed that the value of $(S/N)_0$ given in C would also be applicable in cases where, owing to the time-base velocity and i.f. bandwidth used, the grain of noise is too fine for the resolution of the noise peaks. The difference between the values of $(S/N)_0$ obtained for A and B emphasizes the suggestion about the dominance of the brilliancy criterion in variation A of the experiment.

The differences between the values of σ_1, σ_2 and σ_3 in the three variations are barely significant statistically.

5. Application of the Results

The application of the values of $(S/N)_0$ obtained will now be discussed in greater detail. Firstly, the equivalent noise power of a receiver will be defined. The concept of available power is becoming increasingly used in the specification of receiver noise. To conform with this idea the equivalent

receiver noise power N_e^2 will be defined as the available power at the signal-generator output, which, falling wholly into the receiver band would produce the i.f. output noise power. This does not of course mean the actual available thermal-noise power, N_t^2, from the signal-generator output impedance, and in fact the ratio N_e^2/N_t^2 may be used to define the noise factor of the receiver F. Thus

$$F = \frac{N_e^2}{N_t^2} = \frac{N_e^2}{kTB} \quad .. \quad .. \quad .. \quad (1)$$

where k is Boltzmann's constant, T is the absolute temperature of the signal-generator output impedance and B is the energy bandwidth of the receiver.

The equivalent receiver-noise power is therefore given by

$$N_e^2 = \frac{s^2}{(S/N)_0^2}, \quad .. \quad .. \quad .. \quad (2)$$

where s^2 is the available signal-generator power at visual setting and $(S/N)_0$ is again the reference signal-to-noise ratio at the i.f. output. We may put

$$F = \frac{s^2}{(S/N)_0^2 kTB} \quad .. \quad .. \quad .. \quad (3)$$

Thus if the energy bandwidth of the receiver is known the noise factor can be determined from the visual setting.

The signal generator used for the measurement of receiver noise should have an output impedance equal to that of the aerial with which the receiver is intended to work. In general, therefore, a particular signal-generator output impedance will be required for each type of receiver and aerial. N_e^2 defined as above also applies for a receiver when used with a particular aerial.

When determining the noise factor F according to Equ. (3) the bandwidth of each receiver must be measured. In order to avoid this measurement it has been proposed to use an additional video filter of a known narrow bandwidth in the measuring equipment. The accuracy of this method depends on whether the effect of introducing a narrow video filter of bandwidth, B_V, is identical with that of reducing the i.f. bandwidth to B_V or at least to B_V multiplied by some constant, g say. If it were identical then in Equ. (3) gB_V would replace B. This implies that the available signal power at setting, s_V^2, would be proportional to B_V; i.e., s_V^2 would vary with B_V at the rate of 3 db per octave It is shown in the Appendix that, for linear detection and $B_V \ll B$, the rate of change of s_V^2 with B_V is 1.5 db/oct. The reason for this lower rate of change is that the input signal, and so the i.f. signal-to-noise ratio at setting, is reduced to such an extent that signal suppression by noise occurs at the detector. The rate of 1.5 db/oct means that:

$$\frac{s_V^2}{s^2} = g\sqrt{\frac{B_V}{B}}, \quad .. \quad .. \quad .. \quad (4)$$

and using this relation, Equ. (3) may be written in the form:

$$F = \frac{s_V^2}{(S/N)_0^2 gkT} \cdot \frac{1}{\sqrt{BB_V}} \quad .. \quad (5)$$

Thus the use of severe video filtration cannot eliminate the need for measuring B. It is shown in the Appendix that the rate of 1.5 db/oct also applies to the square-law detector.

When routine measurements are made on one type of receiver the bandwidth of a typical receiver can be determined and used in Equ. (3) for all receivers of the type. An error in F will then arise from the variation in receiver bandwidths. It can be seen from Equ. (5) that this error may be halved by using severe video filtration. In this case it would be necessary to determine the factor g. This can be done in the following way; s_V^2 is found with the video-filter method for one receiver whose bandwidth is known, then with the video filter removed s^2 is measured, finally g may be found from Equ. (4). By using the appropriate value of $(S/N)_0$ given in Section 4 F can then be found from Equ. (5).

6. Conclusions

Three variations A, B, C, of the visual method employing the Army setting were examined. In each case definite verbal instructions were given. In A the operators were instructed, "Keep bottom of top noise level with top of bottom noise" In B the instructions were "Adjust input so that top noise is completely above bottom noise; i.e., so that no peaks overlap." In C the same instructions were given as in A but in contrast to A and B the c.r. tube spot was defocused to prevent resolution of individual noise peaks. The reference values of i.f. output signal to noise ratio at setting were found to be A, 8.3 db; B, 11.2 db; C, 8.8 db.

The reference values of signal-to-noise ratio show that a pronounced difference

can be made by varying the instructions.

The agreement of the values of the reference signal-to-noise ratios found in A and C suggest that the setting is determined not so much by the number of noise peaks reaching any level as by the average brilliancy at this level.

For each variation of the visual method the standard deviation of the error of a single reading was about 1.3 db. The standard deviations of the readings of a single observer was about 0.5 db in each variation.

On the basis of the above conclusions variation A of the method is recommended.

For routine measurements of noise factor on one type of receiver the error due to neglecting the difference between individual receiver bandwidths may be halved by using a narrow-band video filter.

Acknowledgment

The permission of the Chief Scientist, Ministry of Supply, to publish this paper is gratefully acknowledged. The author wishes to thank Mr. Z. Jelonek for the suggestion of the problem and for help received in the preparation of the article.

APPENDIX

The relation between video-filter bandwidth and signal-generator power at visual setting.

For convenience it will be assumed that there is no video amplification and that, for the linear detector, the output voltage is equal to the amplitude of the input voltage.

It has been shown* that the difference between the mean detector output voltage for noise alone and noise with signal, y, is given approximately by

$$y \approx \tfrac{1}{2}\sqrt{\frac{\pi}{2}} \cdot E\,(S/N)^2 \qquad .. \quad .. \quad .. \quad (5)$$

for $(S/N) \ll 1$. (S/N) is again the i.f. output signal-to-noise ratio and E is the i.f. output r.m.s. noise voltage. y is indicated in Fig. 1(b).

It has also been shown* that the r.m.s. detector output voltage about the mean for free noise (i.e., $(S/N) = 0$) is given by

$$v' = E\left(2 - \frac{\pi}{2}\right)^{\frac{1}{2}}, \qquad .. \quad .. \quad .. \quad (6)$$

and for modulating noise the corresponding voltage is, for $(S/N) \ll 1$

$$v'' \approx E\left[\left(2 - \frac{\pi}{2}\right) - \left(2 - \frac{\pi}{4}\right)(S/N)^2\right]^{\frac{1}{2}} \quad .. \quad (7)$$

Since $(S/N) \ll 1$ a further approximation can be made and the term in $(S/N)^2$ in (7) can be neglected, in this case v' and v'' are equal.

Let it be supposed that the i.f. band is rectangular and of width B. Then the power spectrum of free noise after linear detection is very nearly triangular in shape, falling linearly from a maximum value at zero frequency almost to zero at a frequency B. The power spectrum is accurately triangular for a square-law detector. As (S/N) increases, the power spectrum of modulating noise tends to a rectangular shape extending from zero frequency to a frequency $B/2$ for both linear and square-law detectors. Thus, whatever the value of (S/N), we may assume without much error that the power spectrum is constant over a small frequency range. Thus, after transmission through a video filter of bandwidth B_V, the r.m.s. voltages about the means become for free and modulating noise and $(S/N) \ll 1$,

$$V_V' = V_V'' = E\left[\left(2 - \frac{\pi}{2}\right)\frac{B_V}{B}\right]^{\frac{1}{2}} \qquad .. \quad (8)$$

As long as $(S/N) \ll 1$ little error will be made by assuming the spectral density of noise to be constant over a small frequency range near zero for any i.f. pass-band shape. For a shape other than rectangular however, Equ. (8) will be modified by the insertion of a constant factor characteristic of the band shape.

The probability distribution of linearly detected free noise before video filtration is of the Rayleigh type [illustrated in Fig. 1(b)]. After video filtration the form of the distribution is difficult to calculate but it has been shown by S. O. Rice† that for severe video filtration the probability distributions of free and modulating noise approach the Gaussian form. Thus if the visually estimated top (or bottom) of a Gaussian noise distribution is at a level of m times the r.m.s. voltage of the noise from the mean, then at visual setting

$$y = m(V_V' + V_V'') = 2m\left[\left(2 - \frac{\pi}{2}\right)\frac{B_V}{B}\right]^{\frac{1}{2}} E \quad (9)$$

Substituting this value of y in Equ. (5), we have

$$(S/N)^2 = 4m\left[\left(\frac{4}{\pi} - 1\right)\frac{B_V}{B}\right]^{\frac{1}{2}}, \qquad .. \quad .. \quad (10)$$

and since s^2, the available signal generator power at visual setting, is proportional to $(S/N)^2$ we have

$$s^2 \propto \left(\frac{B_V}{B}\right)^{\frac{1}{2}}. \qquad .. \quad .. \quad .. \quad .. \quad (11)$$

Equ. (11) also holds for the square-law detector. This is to be expected since, as is shown by Equ. (5), the linear detector behaves like the square-law detector as far as the signal output, y, is concerned for $(S/N) \ll 1$.

* In an unpublished report by R. E. Burgess, N.P.L. circulated to the R.R.B. "The Rectification of Signal and Noise by Linear and Square-Law Detectors." Paper No. RRB/C.93. 24th March, 1944.

† "The Mathematical Analysis of Random Noise." *Bell System Technical Journal*, 1945, Vol. 24, p. 46.

R.F. GENERATOR LOAD*

Use of Water-Dielectric Transmission Line

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SUMMARY.—The ordinary conventional relations employed in transmission-line theory are used to develop the characteristics of a short-circuited concentric line having a dielectric of ordinary tap water. The relations between line dimensions, water conductivity and input impedance are given, and it is shown that a steady known rate of water flow through the line, together with a measurement of water temperature rise permit the input power to be found. Measured values of line input impedance and experience with the line under loading conditions are given.

Introduction

THE measurement of output power from all sizes of r.f. generators up to frequencies in the region of 20 Mc/s is a comparatively easy matter. Methods of loading the generator include the use of ordinary tungsten or carbon bulbs and water- or air-cooled carbon resistors ; whether lamps or resistors are used a multiplicity of either is necessary for the higher powers.

For large output powers and at higher frequencies the use of lamps or resistors tends to become unsatisfactory. The lamps usually light up unevenly, due to the distributed inductance and capacitance, and ionization often occurs due to the filament-lead inductance. Similarly, carbon resistors heat up unevenly and consequent damage to the carbon surface may result. In addition, the multiplicity of lamps or resistors usually takes up considerable room and connection to the generator circuit may be difficult as it is undesirable to introduce reactance in series with the load.

For these reasons the method described below has been developed. At mains frequencies a common method of loading generators is to employ a brine tank fitted with adjustable electrodes ; the brine acts as the dissipating medium, and the heat is then lost to the atmosphere. At radio frequencies the use of the brine tank is limited to the lower frequencies as the capacitance due to the electrodes causes the load to be considerably reactive. Obviously the reactance could be balanced out, but a simpler solution is the use of the transmission line employing water as the dielectric medium.

The reactance of the water-dielectric line can be made zero but, in actual practice, a line adjusted to a quarter-wavelength or an odd multiple of a quarter-wavelength, in spite of its being slightly reactive, can be tolerated as tuning facilities are usually available on the generator circuit.

The advantage of this system of loading is the compactness resulting from the high dielectric constant of water and the high dissipation capabilities of the line due to the water acting both as a dissipating medium and as a heat carrier, rather than merely as a heat carrier as in the water-cooled carbon resistor.

* M.S. accepted by the Editor, July 1946.

Theoretical Basis

Employing conventional relations, the input impedance of a transmission line filled with an imperfect dielectric and short-circuited at its distant end will be determined.

For any short-circuited transmission line the input impedance is :

$$Z_i = Z_c \tanh (\alpha + j\beta)l \quad .. \quad (1)$$

where Z_c = characteristic impedance of the line (ohms)
α = attenuation constant of the line (nepers/cm)
β = phase-shift constant of the line (radians/cm)
l = line length (cm).

Expanding (1) and rationalizing

$$Z_i = Z_c \frac{\cosh \alpha l \sinh \alpha l + j \cos \beta l \sin \beta l}{\cosh^2 \alpha l \cos^2 \beta l + \sinh^2 \alpha l \sin^2 \beta l} \quad (2)$$

The characteristic impedance of the line is given by

$$Z_c = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad .. \quad .. \quad .. \quad (3)$$

where R = resistance of the line (ohms/cm)
L = inductance of the line (henrys/cm)

C = capacitance of the line (farads/cm)

G = conductance of the line (mhos/cm)

$\omega = 2\pi f$, where f is the operating frequency of the line.

For the materials which might be employed in the construction of the line and at frequencies where the line is likely to be used, $\omega L \gg R$, thus equation (3) may be re-written.

$$Z_c = \sqrt{\frac{L/C}{1 + G/j\omega C}}$$

$$= \sqrt{\frac{L}{C}}\left\{1 + \frac{j}{2}\frac{G}{\omega C} - \frac{3}{8}\frac{G^2}{\omega^2 C^2} + \ldots\right\} \quad (4)$$

For $G/\omega C \leqslant 0.3$ then the above may be written with sufficient accuracy

$$Z_c = Z_0\left(1 + \frac{j}{2}\frac{G}{\omega C}\right) \quad .. \quad (5)$$

where $Z_0 = \sqrt{\frac{L}{C}}$

Substituting equation (5) into equation (2)

$$Z_i = \frac{Z_0}{\cosh^2\alpha l \cos^2\beta l + \sinh^2\alpha l \sin^2\beta l}$$
$$\left\{\begin{array}{l}[\cosh\alpha l \sinh\alpha l - n\cos\beta l \sin\beta l] \\ + j\,[\cos\beta l \sin\beta l + n\cosh\alpha l \sinh\alpha l]\end{array}\right\} \quad (6)$$

where $n = \frac{G}{2\omega C}$

Thus the phase angle of the input impedance is given by

$$\phi = \tan^{-1}\left\{\frac{\cos\beta l \sin\beta l + n\cosh\alpha l \sinh\alpha l}{\cosh\alpha l \sinh\alpha l - n\cos\beta l \sin\beta l}\right\} \quad (7)$$

for $\phi = 0$

$$\cos\beta l \sin\beta l + n\cosh\alpha l \sinh\alpha l = 0$$

that is,

$$\sin 2\beta l + n\sinh 2\alpha l = 0 \quad .. \quad .. \quad (8)$$

The solution of equation (8) permits the value of l giving unity power factor to be found.

It is interesting to note that the phase angle of the input impedance when l is a quarter-wavelength is

$$\phi = \tan^{-1} n \quad .. \quad .. \quad .. \quad (9)$$

and that the input impedance is

$$Z_i = Z_0\,(1 + jn)\coth\alpha l \quad .. \quad .. \quad (10)$$

The attenuation and phase-shift constants will now be determined. The attenuation constant is given by

$$\alpha^2 = \frac{1}{2}\left[\sqrt{(R^2 + \omega^2L^2)(G^2 + \omega^2C^2)} + RG - \omega^2LC\right]$$

all the above symbols have already been defined. The above may be re-written

$$\alpha^2 = \frac{1}{2}\left[\sqrt{\omega^2L^2(G^2 + \omega^2C^2)} - \omega^2LC\right] \quad (11)$$

as $\omega L \gg R$ and $\omega^2LC \gg GR$ for reasons already given in connection with equation (4).

Equation (11) may be expanded giving

$$\alpha^2 = \frac{1}{2}\omega^2LC\left\{\frac{1}{2}\left(\frac{G^2}{\omega^2C^2}\right) - \frac{1}{8}\left(\frac{G^2}{\omega^2C^2}\right)^2 + \frac{1}{16}\left(\frac{G^2}{\omega^2C^2}\right)^3 - \ldots\right\} \quad .. \quad (12)$$

For $G/\omega C \leqslant 0.3$ the attenuation constant may be written

$$\alpha = \frac{1}{2}\sqrt{\frac{L}{C}} \cdot G = \frac{1}{2}Z_0G \quad .. \quad .. \quad (13)$$

The phase-shift constant is given by

$$\beta^2 = \frac{1}{2}\left[\sqrt{(R^2 + \omega^2L^2)(G^2 + \omega^2C^2)} - RG + \omega^2LC\right] \quad .. \quad (14)$$

As for the attenuation constant, the above may be re-written

$$\beta^2 = \frac{1}{2}\omega^2LC\left\{2 + \frac{1}{2}\left(\frac{G^2}{\omega^2C^2}\right) - \frac{1}{8}\left(\frac{G^2}{\omega^2C^2}\right)^2 + \ldots\right\} \quad .. \quad (15)$$

For $G/\omega C \leqslant 0.3$ the phase shift constant may be written

$$\beta = \omega\sqrt{LC} = \frac{2\pi\sqrt{\kappa}}{\lambda_0} \quad .. \quad .. \quad .. \quad (16)$$

Where λ_0 = the free-space wavelength (cm)

κ = the dielectric constant of the medium surrounding the lines ($\kappa = 1$ for air).

The Practical Line

A concentric line will now be considered; for balanced loading it is thought preferable to employ two concentric lines mechanically connected, rather than a balanced shielded-pair, as the water heating in this latter case would be much less uniform than for the concentric lines.

The value of Z_0 for a concentric line is given by

$$Z_0 = \frac{138}{\sqrt{\kappa}}\log_{10}\frac{r_2}{r_1} \quad .. \quad .. \quad (17)$$

Where r_2 = radius of the outer conductor

r_1 = radius of the inner conductor.

Taking the dielectric constant for water as 81, equation (17) can be written

$$Z_0 = 15.4 \log_{10} \frac{r_2}{r_1} \quad .. \quad .. \quad (18)$$

The value of G is determined by the volume resistivity and the dielectric loss of the water; it is assumed that for tap water the dielectric loss is very much less than that due to the volume resistivity and, therefore, can be ignored.

This being so, $G = \frac{2\pi}{\rho \log_e r_2/r_1}$ mhos/cm .. .. (19)

where ρ is the volume resistivity in ohms/cm[3].

The value of C for the line is

$$C = \frac{\kappa}{2 \log_e r_2/r_1} \cdot \frac{10}{9} \cdot 10^{-12} \text{ farads/cm} \quad .. \quad .. \quad (20)$$

Thus from (19) and (20)

$$G/\omega C = \frac{2\pi}{\rho \log_e r_2/r_1} \cdot \frac{18 \times 10^{11} \log_e r_2/r_1}{2\pi f \kappa} = \frac{18 \times 10^{11}}{\rho f \kappa} \quad .. \quad .. \quad .. \quad (21)$$

Taking the dielectric constant of the water as 81, equation (21) becomes

$$\frac{G}{\omega C} = \frac{2.22 \times 10^{10}}{\rho f} \quad .. \quad .. \quad .. \quad (22)$$

From (13)

$$\alpha = \frac{1}{2} \cdot \frac{60}{\sqrt{81}} \cdot \log_e \frac{r_2}{r_1} \cdot \frac{2\pi}{\rho \log_e r_2/r_1} = \frac{21}{\rho} \quad (23)$$

From (16)

$$\beta = \frac{18\pi}{\lambda_0} \quad .. \quad .. \quad .. \quad .. \quad (24)$$

Consideration will now be given to an actual line having the following dimensions and operating frequency:

$r_2 = 25.5$ mm
$r_1 = 16$ mm
$\rho = 3{,}800$ ohms per cm cube. (Result obtained by measurement on a sample of tap water.)
$f = 30$ Mc/s ($\lambda_0 = 1{,}000$ cm).

Thus

$$\frac{G}{\omega C} = \frac{2.22 \times 10^{10}}{3.8 \times 10^3 \times 30 \times 10^6} \approx 0.2$$

$$Z_0 = 15.4 \log_{10} \frac{25.5}{16} = 3.1$$

Hence

$$Z_c = 3.1(1 + 0.1j)$$

From equation (23)

$$\alpha = \frac{21 \times 10^{-3}}{3.8} = 5.52 \times 10^{-3}$$

From equation (24)

$$\beta = 3.24°/\text{cm}.$$

Suppose the line is a quarter-wavelength long; i.e.,

$$l = \frac{90}{3.24} = 27.8 \text{ cm}$$

from (10) the input impedance is

$$Z_i = 3.1(1 + 0.1j) \coth(27.8 \times 5.52 \times 10^{-3}) = 20.4\ (1 + 0.1j) = 20.6\ \underline{/5.7°} \text{ ohms}.$$

Two mechanically connected concentric lines, short-circuited at one end and having the above dimensions were constructed, the double construction permitting either a balanced or unbalanced load to be obtained. The material used was telescopic brass tubing, thereby enabling the line length to be adjusted for use at various frequencies. An outline sketch of the arrangement is shown in Fig. 1.

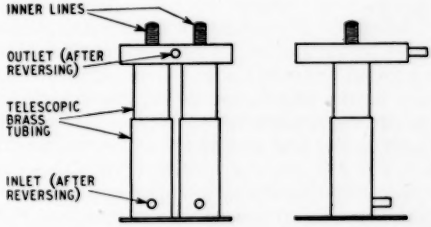

Fig. 1. *This sketch shows the arrangement of the double concentric-line load constructed from brass tubing.*

Temperature measurement of the water inlet and outlet is made some 18 inches from the line as this permits the thermometers to be well free from screens, etc., which may surround the generator circuits. Ordinary $\frac{3}{4}$-inch rubber hose is used for connecting to the water supply and the waste pipe.

It is particularly important that during operation no air remains in the line. With the arrangement described it was found difficult to eliminate all the air; however, merely by changing over the inlet and outlet water pipes this difficulty was overcome. If air exists in the line then very probably the water flow will give rise to a rushing noise. Another test for air is to observe the generator grid and anode current as the water flow is gradually increased or decreased; as the air volume changes due to the changing water flow, so the currents vary.

Tap water in contact with brass results in

very slight corrosion, but the rate at which this occurs is very slow. At increased temperatures precipitation of dissolved salts may occur ; however, as the temperature of the brass can easily be maintained low, no trouble should be experienced due to this latter effect.

Thermometers of the Beckman type can very well be used where higher accuracy is required, but ordinary 50° C range thermometers are quite suitable for general test use. Thermocouples in conjunction with a galvanometer are not considered suitable, as screening becomes rather difficult where high powers are concerned. Heat exchange between the line and the air will take place unless the line is lagged. However, in most cases the generator output will be high and the heat exchange can very well be ignored, especially if the rate of water flow is adjusted so that the temperature rise is only a few degrees.

The input impedance of one line at unity power factor and at a frequency of 28.6 Mc/s was found to be 19 ohms, which is reasonably close to the calculated input impedance for a quarter-wave line at 30 Mc/s. The overall length of the line was 33 cm ; the real length of the line is slightly different from this due to the discontinuity at the open end. Because of the low input impedance of the line it is particularly necessary to make the connecting leads to the generator circuit of very low inductance, and in the experiments the line was attached to the generator circuit by means of copper strip 1.5 inches wide and with a lead length of about 0.25 inch.

Output measurements at 30 Mc/s and 70 Mc/s respectively were made as follows. At 30 Mc/s the line was adjusted to approximately a quarter-wavelength, power was obtained from a push-pull earthed-anode oscillator using balanced transmission lines as circuit elements. The output power, as determined from the water flow and temperature rise, was 5 kW. This output power was dissipated quite satisfactorily. If the input resistance of one line at unity power factor and 30 Mc/s is 19 ohms, then with 5-kW input the voltage between the inner and outer conductors of each line is only approximately 220 volts r.m.s. and the current at the short-circuited end very nearly 70 amperes r.m.s. The maximum dissipation for the brass surface would therefore only be approximately 0.15 watt/cm², and hence no trouble can be expected due to high temperature of the brass. The line voltage is low and suitable design of the support insulators for the inner conductor should exclude the possibility of electrical breakdown. If the water flow is such that only a small temperature rise occurs, then it is considered that powers considerably in excess of the above can be dissipated. At 70 Mc/s the line was adjusted to approximately three-quarter wavelengths and a balanced earthed-anode oscillator was again used. The output power as determined from the water flow and temperature rise was 670 watts. The oscillator output under the same operating conditions was determined using a multiple lamp load and the output was found to be 655 watts, thus giving very satisfactory agreement between the two methods of power measurement.

Acknowledgment

The author wishes to acknowledge his thanks to the Mullard Radio Valve Co. for permission to publish the paper, and also to several members of the Research and Development Department for helpful suggestions.

TRANSIENT RESPONSE OF V.F. COUPLINGS

Two errors occurred in the article by W. E. Thomson in the January issue. On p. 21, col. 2, in the expression for G read $1 + (1/p)$ instead of $(1 + 1/p)$, and on p. 22, col. 2, the second expression for V should read

$$V = \frac{1}{\alpha - 1}\left(\alpha e^{-t/\alpha T} - e^{-t/T}\right)$$

It should also be pointed out that on p. 26 in the second line the superscript (2) is not an index of p, but a reference number.

INSTITUTE OF PHYSICS

The annual general meeting of the Electronics Group of the Institute of Physics will be held in the Rooms of the Royal Society, Burlington House, Piccadilly, London, W.1., at 5.30 p.m. on the 22nd April. It will be followed by a lecture on " Electron Multipliers " by S. Rodda.

BOOK RECEIVED

Reference Data for Radio Engineers (2nd Edition) Compiled by W. L. McPherson, B.Sc., M.I.E.E., S.M.I.R.E. Pp. 175. Published by Standard Telephones and Cables, Ltd., Connaught House, 63, Aldwych, London, W.C.2. Price 5s. (Postage 6d.).

Originally compiled for use within the Standard Telephones and Cables organization, this book is now generally available and contains a large amount of reference material. It ranges from general engineering tables to mathematical formulae and tables, and includes data on materials, rectifiers, propagation, aerials and waveguides.

CAVITY RESONATORS AND ELECTRON BEAMS

By J. H. Owen Harries, A.M.I.E.E., M.I.R.E.

(*Continued from p. 80, March 1947*)

9. A Physical Explanation of the Analysis

THE devices analysed in this paper consist of three main elements :—

(a) A modulated beam of electrons. (The modulation is assumed to be 100 per cent).
(b) A resonant cavity.
(c) A load.

The modulated electron beam is arranged to travel into the internal field of the resonant cavity so that part of the kinetic energy of the electrons is changed into electromagnetic wave energy inside the cavity. The efficiency (η_0) at which this change takes place would be 100 per cent if all the kinetic energy of the electrons in the beam were changed into electromagnetic wave energy inside the resonator ; but η_0 has, in practice (with a sinusoidal 100 per cent modulation of the electron beam current) a best possible theoretical value of about 43 per cent (Fig. 2) and this is attained only if the two following requirements are met :—

(i) First, the entrance voltage of the electrons into the resonator field and the time they take to travel through the field as compared to the periodic time of the oscillations of the field, must together be such that the so-called "small-signal" transit angle ϕ has a value of about $\phi_0 = \pi/2$ [Equations (14) and (15)].

(ii) Secondly, the ratio M (between the peak value of the voltage V'_R along the path of the electron beam through the internal field of the resonator and the steady voltage V_B at which the electrons enter the resonator) must be rather greater than unity. Quantitatively, we must have $M = M_0 = 1.24$, (Equation(14) and Fig. 2).

It may make this clearer if it is pointed out that the corresponding requirement for full efficiency in an "ordinary" triode-valve amplifier, working at a low frequency, is the familiar one that the steady anode voltage must be as nearly as possible fully modulated by the peak alternating voltage across the load. That is, M should equal $M_0 = 1$ when the transit angle ϕ is negligible. This is a special case of the general analysis set out in this paper.

The voltage V'_R itself depends, of course, upon the total amount of power (P_R) delivered to and dissipated in the resonator, and upon the field distribution inside the resonator at the mode of resonance used. This leads to the use of a parameter which is characteristic of the resonator itself, and which has been designated by the symbol ζ. It is equal to $V'^2_R/2P_R$ [equation (30)]. It has the dimensions of resistance.

(The familiar "anti-resonant impedance" L/CR of a tuned lumped-constant L/C circuit is a special case of the general parameter ζ).

In addition to the power P_R delivered to the resonator, an output power P_L must be simultaneously delivered to the load.

A power P_0 must therefore be delivered from the electron beam which is equal to $(P_L + P_R)$ [equation (8)].

The load must be coupled to the resonant cavity in such a way that the power P_L delivered to it is much greater than the power P_R lost in the resonator [equation (10)]. That is to say, if the efficiency η_C of the transfer of power from the resonator to the load is to be good (say 90 per cent), then the "selectivity factor" Q of the resonator when unloaded must be 10 times the selectivity factor Q_L when it is loaded [equation (11) and Fig. 5].

The overall efficiency η is clearly the product of the efficiency η_0 of conversion of the kinetic energy of the electrons in the beam into electromagnetic-wave energy, and the efficiency η_C of transfer of electromagnetic-wave energy from the resonator to the load [equation (17)].

All the somewhat complicated requirements set out above must be simultaneously satisfied if the overall efficiency η is to be as high as possible. It is necessary in any

given case, then, to compute the values of the various quantities concerned which will give this result.

This means that a parameter must be found which is common to the (so-far) separate relationships which specify respectively the electron beam and the resonator—load combination. A single common expression for their co-operation may then be written down.

To solve this problem, it is pointed out in the present paper that, in any given shape of resonator, for a given mode and for a given electron-beam path through that resonator, there is only one value of the steady voltageV_B (out of all those which can be used to accelerate the electrons into the resonator) which will cause the "small-signal" transit angle ϕ to have the necessary optimum value ϕ_0 of about $\pi/2$.

This one special voltage has been indicated herein by V_{OB}, and called the "characteristic voltage of the resonator," for a given mode and beam path, because it is the only voltage with which it is possible to obtain the required efficiency η_0 of transfer of energy from the beam to the oscillating field inside that particular resonator when it is working at a certain mode and with a certain beam path therein.

V_{OB} is that common parameter which is needed to link the electron-beam equations and the resonator—load equations [equation (22)]. This is because V_{OB} is both a special value of the steady voltage V_B which accelerates the electrons into the resonator, and, in the special circumstances set out, is also equal to 1.24 times the peak oscillatory voltage V'_R delivered inside the resonator along the path of the electrons.

Bearing in mind that the resonant wavelength of any resonator varies linearly with its size, it will be seen, from equation (15), that the "characteristic voltage" V_{OB} is independent of the size and resonant wavelength of the resonator.

These considerations are those which lead to the equations [(e.g. (37) and (38)] for η_0 and P_L set out in the preceding part of this paper.

Attention must be drawn to an important corollary.

The power P_0 delivered from the beam to the electromagnetic field is $\eta_0 I_0 V_{OB}$. η_0 and V_{OB} are, in a given case, constants. The power P_0 supplied to the field is therefore proportional to the beam current I_0.

If the efficiency η_0 of the transfer of some of the power $I_0 V_{OB}$ in the electron beam to the resonator is to be at the best value, the power P_0 supplied to the internal field of the resonator from the beam must be equal to the sum of the power P_L used in the load plus the power P_R which has to be delivered to the resonator in order to produce a peak voltage V'_R along the beam path of 1.24 times $V_B = V_{OB}$. The amount of power P_R required for the latter purpose is calculable if the value of $\zeta = V'^2_R/2P_R$ is known for the resonator shape, mode and beam path in question.

Remembering that, in a given case, V_{OB} is a constant, it follows that the "resistance" $V_B/I_0 = V_{OB}/I_0$ of the beam (indicated by R_B in the present paper) must be arranged to have a certain definite value, for any given resonator and load combination, if the beam-to-resonator field efficiency η_0 is to be at its best possible value. At the same time, the efficiency η_C (of transfer of power from the resonator field to the load) must be given as high a value as possible by arranging that the ratio between the power P_L delivered to the load and the power P_R dissipated in the resonator is as great as possible.

Therefore, in a given valve and resonator structure, and therefore a given value of V_{OB}, there are two parameters available for adjustment to obtain the maximum overall efficiency η; namely, the value of the beam current I_0 and the value of the coupling coefficient between the load and the resonator which determines the ratio between P_L and P_R. Since P_R is fixed for a given resonator and characteristic voltage V_{OB}, the latter adjustment controls the total power P_0 which has to be delivered from the beam to the resonator for V'_R to equal 1.25 V_{OB}, and for η_0 to be at its best value, as well as determining the resonator-to-load efficiency η_C.

For any given kind of valve (klystron or magnetron, for instance), and for a given geometrical size, the ratio $V_{OB}/I_0 = R_B$ cannot (even theoretically) be reduced below a certain value, because it is inherently impossible to pack into a given space more than a certain number of electrons which are moving at a given velocity. In practice, one can do no more than approach as closely as one can to this inherent limit, because it is set by the unalterable charge and mass of electrons. The consequent lower limit to R_B is, of course, much higher for valves which use narrow and confined

beams of electrons than for those which use diffused beams from a large area of cathode. But a limiting value exists in all cases (Bibs. 15 and 16), and, unfortunately, it does not lie at very low values of R_B. For example, of the various practical electrode shapes, that of the cylindrical diode has clearly by far the lowest R_B values; but a typical 10-cm wavelength cavity-type magnetron (of this general shape) has an R_B of as much as 3,000 to 800 " ohms " per cavity. Valves, such as klystrons, which use a narrowly confined beam of electrons, commonly run into four or even five figures for this quantity.

In illustration of the effect of all this, we have the following possible conditions:—

(*a*) When the parameter $\Psi = \frac{\zeta}{R_B}$ [equations (29) and (37)] is high enough for V'_R to be made equal to 1.25 V_{OB}, with an attainable value of $I_0 V_{OB}$ for the valve in question, although P_R is (as it should be) small compared with the power in the load P_L. In this case, clearly, the beam-to-resonator field efficiency (η_0) is at its best value, and the resonator-to-load efficiency η_c is usefully high. That is $\eta_0 \approx 43$ per cent and $\eta_c \rightarrow$ 100 per cent. Therefore, the overall efficiency η is good. One may regard the resonator-load in combination as " matched " correctly to the electron stream.

(*b*) When Ψ is not high enough, with any attainable value of $I_0 V_{OB}$, for V'_R to equal 1.25 V_{OB}, unless the coupling coefficient from the resonator to the load is reduced to such an extent that P_L is of the same order as, or even less than, P_R. In this case the resonator—load combination can be regarded as capable of being correctly " matched " to the electron stream; but the resonator-to-load transfer efficiency η_c is low if this "matching" is done. For all possible adjustments the overall efficiency η will be comparatively bad.

(*c*) When Ψ is so low that there is insufficient power $I_0 V_{OB}$ available from the beam for V'_R to equal 1.25 V_{OB} in any circumstances, even though the coupling coefficient from the resonator to the load is reduced to a negligible value. The beam-to-resonator efficiency η_0 will be low and the resonator-to-load efficiency η_c cannot be made high without reducing η_0 still more. The overall efficiency $\eta = \eta_0 \eta_c$ will clearly be very bad indeed.

Because the available range of R_B is generally severely limited by practical considerations of valve design, as well as by the theoretical limit to the possible electron current flow I_0 in a given geometrical space, the most important practical problem becomes that of making ζ as high as possible.

It has been shown [equation (30)] that ζ is equal to the product of the familiar selectivity factor Q of the unloaded resonator and of a quantity the writer has called the " voltage coefficient " ξ [equation (18)]. For simple shapes of cavity and for known materials, Q and ξ may be computed. Alternatively, methods of measuring these quantities are available and are set out in a later part of this paper.

The voltage coefficient ξ is found to have serious limitations to its magnitude. One hundred or so appears to be the limit even in very favourable resonator shapes. The limitations to the value of Q are only too familiar even when the resonator is made of material of the highest known conductivity; namely, pure polished copper.

The problem of improving $\zeta = \xi Q$ in a resonator is much the same as the familiar problem of getting as high as possible a value to the L/CR value of a long-wave "lumped inductance—capacitance" resonant circuit. The L/CR ratio of this circuit is indeed a special case of the general equation for ζ. The highest possible values of ζ for a resonator (like the highest values of L/CR for a lumped circuit) are difficult to obtain.

An ultimate theoretical and, therefore, rigid, upper limit to ζ must, in fact, exist for the following reasons. No better conductor than pure copper is known; the shape of the cavity is determined by the inherent properties of electromagnetic waves; inherent geometrical limits exist to the length of the beam path in the resonator (to give the optimum " small signal " transit angle $\phi = \phi_0 = \pi/2$) and the cross-sectional area A_0 of the electron beam has a necessarily finite value. Note that the last two statements infer that some part of the volume of the resonator must be arranged so that the electric field in that part is substantially uniform. (It is of course possible to get a better conductivity than that of pure copper at normal temperatures by, for example, super-cooling a resonator with liquid helium.)

The writer is not aware of any general expression for the theoretical limit to ζ. Nevertheless sufficient practical measurements and calculations have been made to

lead to the conclusion that the ζ value of a resonator at normal temperatures (like the L/CR value of a " lumped constant " circuit) can only reach the order of a megohm or two with some difficulty at any wavelength, and, in practical conditions, is often far less than this.

Therefore, the design of a good electron-beam resonator valve which is intended to deliver electromagnetic-wave energy to a load involves two main problems. First, the problem of electron-stream production and modulation, which must include the inherent limitations of materials as regards cooling. (Copper has the greatest heat conductivity as well as the greatest known electrical conductivity at normal temperatures.) Secondly, the problem of trying to design a resonator the ζ value of which is as high as possible in comparison with the electron-stream " resistance " R_B.

The choice of the actual electronic and resonator-load designs, to be used respectively to solve these two problems, will depend on many non-analytical matters, and primarily upon the wavelength range over which the device is to be worked. Therefore, in practice, the various parameters ζ, R_B have to be specified for that particular wavelength range, and this leads to the use in this paper of the concept of a " reference wavelength " λ_0, and to the corresponding parameters ζ_0, R_{B0}, Q_0, etc., which refer to operation at that reference wavelength [Equations (37), (38), and (39)].

10. Wavelength Limitations of Electron-Beam Valves and Circuits.

When one endeavours to design an electron beam-resonator valve for operation at extremely short micro-wavelengths, one finds it increasingly difficult, as the wavelength is reduced, to attain the values of Ψ one needs. Efficiencies, therefore, tend to become very low. So great is this difficulty that the writer began some time ago to suspect that the trouble might not be merely the transient one of learning a new technique, but that it might, perhaps, be due to the existence of a limit in wavelength beyond which the unalterable properties of materials, electrons and electromagnetic waves themselves, would not permit further progress to shorter wavelengths. It follows from the preceding analysis that such a theoretical limit in wavelength does in fact exist.

The linear dimensions of a resonator are proportional to wavelength, and its physical size determines the power P_0 that can be put into it. The available power in the beam of electrons tends, therefore, to vary as λ^2. Therefore R_B tends to vary as λ^{-2}.

This fact, in conjunction with the existence of an upper limit to the value of ζ, and the fact that ζ tends to vary as $\lambda^{\frac{1}{2}}$, then makes it clear that, as wavelength is decreased, R_B will tend to rise substantially, whereas ζ will not. This results in a progressive reduction in the value of Ψ as the operating wavelength is decreased. Eventually a wavelength must be reached which is so short that even the best obtainable value of Ψ is too low for V'_R to equal 1.25 times V_{OB} (so that η_0 is equal to the best value) without the ratio between the power P_L in the load and the power P_R lost in the resonator (which ratio determines η_C) having to be reduced. Finally, at still shorter wave-lengths, both η_0 and η_C will become so small that the overall efficiency η will become negligible; and thus the familiar combination of a modulated electron stream with a resonant cavity will be useless at these wavelengths.

The quantitative relationships determining this short-wave limit have been computed from the equations of this paper.

Computation has been made for values of Ψ which include a value assumed to be the best possible, as well as values within the range found in practical apparatus.

The best possible value of ζ_{10} (at $\lambda_0 =$ 10 cm) has been assumed to be about 2×10^6. The values of $R_{B0} = R_{B10}$ (also at $\lambda_0 =$ 10 cm) have been taken as covering a range of about 400 " ohms " to about 40,000 " ohms." Dividing these values of R_{B10} into $\zeta_{10} =$ 2,000,000 gives a range of values of Ψ_{10} from 50 to 5,000 [Equation (37)].

In justification of these figures, as already pointed out, magnetrons (because a roughly cylindrical anode is used which surrounds the cathode) have undoubtedly the best R_B values at the shortest micro-wavelengths; but to produce even a cavity magnetron having R_{B10} much less than 400 " ohms,"—one having, for instance, an effective electron current of more than 2.5 amperes at 1,000 volts to each resonator cavity at 10-cm wavelength—appears to be an improbable project indeed. Therefore, a value of $\Psi_{10} =$ 5,000 is hardly likely to be bettered by known means. Indeed it is perhaps rather an over-optimistic figure.

The focused beams of electrons which are used for velocity-modulation systems have values of R_{B10} which are in tens of thousands as a rule, and Ψ_{10} is, therefore, correspondingly less in these valves than in magnetrons. The order of Ψ_{10} would seem

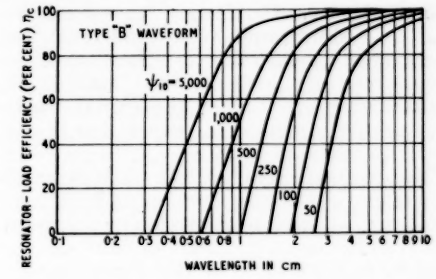

Fig. 6. *The wavelength limitations of present electron-stream-resonator technique. Resonator-load efficiency η_C is plotted as a function of wavelength λ.*

for klystrons to be in the neighbourhood of 100 or so at best. Thus the not inconsiderable engineering advantage of these latter valves may be offset, in some applications, by the better Ψ_{10} values obtainable with magnetrons.

Assuming that the electron beam-to-resonator field efficiency η_0 is required to be maintained at about the best value—that is, the conditions of $V'_R = 1.25\ V_{OB}$, and $V_B = V_{OB}$, are to hold—it is possible, from equation (38) to plot the resonator-to-load efficiency η_C against wavelength in centimetres for the chosen range of Ψ_{10}; including the assumed best possible value of 5,000, which is derived from the best possible values of ζ_{10} and R_{B10} which are, in turn, determined by the unalterable properties of materials, electrons and electromagnetic fields. This plot is shown in Fig. 6, and η_C will be seen to fall catastrophically with decreases of λ around 1 cm wavelength.

From equation (39) it is possible to plot the maximum power P_L which can be delivered to a load for the chosen values of Ψ_{10}, assuming that η_0 is maintained at the best value. Fig. 7 (*a*) shows the results of such a plot for Ψ_{10} equal to the moderate value of 100. Fig. 7 (*b*) shows the result obtained with the assumed best possible value of $\Psi_{10} = 5{,}000$.

The points at which the resonator-to-load efficiency η_C falls respectively to 95 per cent and 60 per cent are marked on these graphs.

Note that the d.c. heat loss $P_B - P_0$ will be very great at the higher ranges of V_{OB}, and will mean that these ranges can only be used for the generation of energy in short-duration pulses so that the average power loss is reduced.

In interpreting Fig. 7 (*a*) and (*b*), note that, although the power output P_L can be increased at very short wavelengths by the artifice of using several resonators in parallel, the result is merely that the curves are displaced vertically by some factor. The slope of the curves is unaffected because the

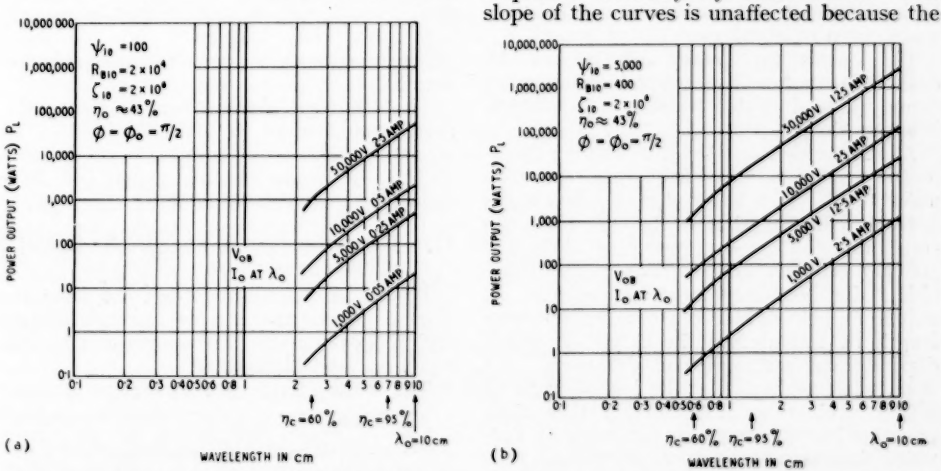

Fig. 7. *The wavelength limitations of present electron-stream-resonator technique. Power output P_L is plotted as a function of wavelength λ. (a) is for $\Psi_{10} = 100$ and (b) for $\Psi_{10} = 5{,}000$.*

artifice can be used at the longer wavelengths as well as at the shortest. The general conclusions are unaffected.

Even if ζ_{10} could be increased several times over the 2,000,000 which is assumed to be the best value possible (and there is no evidence at present that it could be so improved unless one goes to such extreme lengths as to use a super-cooled resonator), no appreciable improvement would be obtained at the shortest-wavelength end of the gamut. This is because the form of equations (38) and (39) is such that the power P_L in the load and the efficiency η_0 are both primarily determined at the shorter end of the wavelength range by the value of the wavelength itself, and only to a comparatively small extent by the other parameters.

With regard to the figures given for the beam current I_0 in Fig. 7, it must not be forgotten that this is assumed to be a fully modulated current. In actual valves the beam current is not fully modulated. In klystrons, for instance, the depth of modulation appears always to be far from 100 per cent. Theoretical figures for "bunching" are not, as far as the author is aware, ever realized in practice, partly because these theories do not appear to take into account "electron-proximity" effects.

All this leads to an important conclusion. The wavelength gamut for the operation of electron-beam resonator valves with a good overall efficiency (i.e., with $\eta_0 \approx 43$ per cent and $\eta_0 \rightarrow 100$ per cent) is found to extend little (if at all in practical cases) below a wavelength of about one centimetre.

The maximum power which can be delivered to a load by such valves falls very rapidly indeed as this boundary wavelength is approached.

At shorter wavelengths than about 1 centimetre, a good overall efficiency η cannot be maintained. If a lesser overall efficiency η is to be tolerated (that is, the case when the conditions that $\eta_0 \approx 43$ per cent and $\eta_0 \rightarrow 100$ per cent are not simultaneously realizable), the boundary may be regarded as extending below about one centimetre wavelength by an interval the extent of which depends on practical considerations, and upon what perhaps may be looked upon as rather problematical factors, rather than upon analytically tractable parameters. Provided that very low output powers and poor overall efficiencies can be tolerated, this extension of the boundary may perhaps be looked upon as covering "millimetre wavelengths."

It is of interest to note that recent information on wave propagation (Bib. 18) shows that difficulties are met in propagating radio waves reliably over reasonable ranges if those waves approach a wavelength of one centimetre. It is a singular coincidence that rain storms and other meteorological phenomena should attenuate these waves so very seriously when the wavelength approaches that part of the gamut where the accepted method of generation is also found to reach its limit.

The method considered in this paper of generating electromagnetic waves by accelerating a modulated beam of electrons through the electromagnetic field of an electric circuit, so that some of the kinetic energy of the electrons is transferred to that field, is the only really practicable one yet known for use over the gamut of radio wavelengths to which it is now applied.

For theoretical, as well as practical reasons, it is difficult to see how any other method of causing electrons to provide electromagnetic energy could compete with this method over that gamut. There are, of course, many other engineering applications of electromagnetic energy at much shorter wavelengths; but the techniques are not those which are the business of radio engineers as such.

The writer is therefore of the opinion that radio engineering is limited to a frequency gamut which may be looked upon as extending from zero frequency up to about 30,000,000,000 cycles per second (1-cm wavelength).

The technical implications of this statement would seem to justify debate. The economic and social implications are not within the scope of this paper, but have been referred to elsewhere.*

11. The parameters of analytically tractable shapes of resonator.

It will be helpful to apply the foregoing theory to some of those few resonator shapes and modes for which the integrals in (34) can be stated explicitly.

The walls of the cavities will be assumed

* See a written contribution by the present author to the discussion on the papers on valves read by Dr. Griffiths and others on the 28th March, 1946, at the Institution of Electrical Engineers' Radiolocation Convention.

to be of copper having a conductivity of 5.8×10^7 mhos/metre. The skin depth δ is then obtainable from Fig. 1.

There is, unfortunately, a lack of agreement among different authors as to the co-ordinates and mode conventions to be used. For the purposes of this paper, all resonators will be considered as sections of waveguides which are terminated by a boundary wall at a distance z_0 along the z dimension. The waves are considered to be propagated along this dimension. According to the convention to be adopted, a resonator will be referred to as operating at an "E" (or TM) mode when it has a component of electric force in the z dimension, and will be referred to as operating in an "H" (or TE) mode if it has a component of the magnetic force in this direction. This convention has, of course, a physical meaning when we are referring to the waves travelling in the z direction down a waveguide. In resonators it is somewhat artificial; but will nevertheless be found convenient.

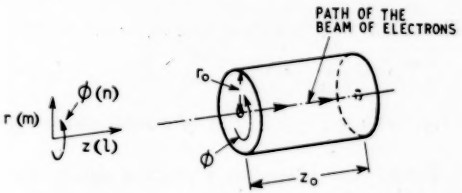

Fig. 8. *An electron-beam path used with a cylindrical resonator. The load is not shown.*

11.1. *Cylindrical cavity resonating at the lowest frequency (E_{010} mode).*

The systems of co-ordinates and suffixes are shown in Fig. 8.

The resonator is assumed to be operating in the E_{010} mode. The electric field is then wholly in the axial or z direction. The beam of electrons must therefore travel in this direction. Because the electric field is at its maximum along the axis, the beam path is chosen to lie along that axis so that the integral V'_R [equation (13)] is at the greatest possible value as is necessary to make $\zeta = V'^2_R/2P_R$ have the highest value possible.

It can be shown (Bib. 4) that in these circumstances the electric field is given by

$$E = E'_z J_0(r'_{nm} r/r_0) \text{ (volts/metre)} \quad (40.1)$$

where

E'_z = the maximum instantaneous value of the electric field.

$J_0(r'_{nm} r/r_0)$ = a Bessel function of the first kind and zero order.

r'_{nm} = the first (mth) root of $J_0(z) = 0$; It is equal to 2.405.

The problem is to solve the integrals in equation (34) or (35) for this resonator shape and mode, and so apply the present analysis.

It may be shown (Bib. 3) that :—

$$Q = \frac{2.63 \times 10^5 r_0}{\lambda_0^{\frac{1}{2}}(1 + r_0/z_0)} \quad .. \quad .. \quad (40.2)$$

where λ_0 = the resonant (and reference) wavelength in metres.

Also,

$$\lambda_0 = 2.61r \quad .. \quad .. \quad .. \quad (40.3)$$

The beam path must be chosen so that V'_R is at its maximum. It will then lie along the axis of the cylinder in the z direction (Fig. 8).

From (40.1) and (13)

$$V'_R = E'_z z_0 \quad .. \quad .. \quad .. \quad (40.4a)$$

and, from (20),

$$z_0/\lambda_0 = a = 5 \times 10^{-4} V_{OB}^{\frac{1}{2}} \quad .. \quad (40.4b)$$

Therefore, in (34)

$$V'_R = 5 \times 10^{-4} V_{OB}^{\frac{1}{2}} E'_z \lambda_0 \quad .. \quad (40.5)$$

The axial dimension of the resonator is therefore controlled by a choice of the characteristic voltage V_{OB}.

The denominator of ξ is $4\pi f$ times the energy W_F stored in the field.

In this case, then,

$$2\omega W_F = \frac{2\pi c \epsilon_0}{\lambda_0} \int E'^2 \, dv$$

$$= \frac{2\pi c \epsilon_0 E'^2_z}{\lambda_0} \int_{\phi=0}^{\phi=2\pi} \int_{r=0}^{r=r_0} \int_{z=0}^{z=z_0} r J_0^2(r'_{nm} r/r_0) \, d\phi dr dz \quad .. \quad .. \quad .. \quad (40.6)$$

From the Lommel integral for $\nu = 0$ (Bib. 6 (16) p. 96).

$$\int r J_0^2(r'_{nm} r/r_0) \, dr = \frac{r^2}{2}[J_1^2(r'_{nm} r/r_0) + J_0^2(r'_{nm} r/r_0)]$$

For $r = r_0$, we have $J_0^2(r'_{nm} r/r_0) = 0$, and

$$\int_{r=0}^{r=r} r J_0^2(r'_{nm} r/r_0) \, dr = \frac{r_0^2}{2} J_1^2(r'_{nm}) = \frac{0.52^2 r_0^2}{2}$$

Therefore, the second integral in (40.6) becomes $0.135 \, r_0^2$. The first integral is 2π, and the third is z_0.

Therefore (40.6) becomes

$$2\omega W_F = \frac{5.4 \times 10^{-1} \pi^2 c \, \epsilon_0 z_0 r_0^2 E'^2_z}{\lambda_0} \quad .. \quad .. \quad (40.7)$$

and from (40.3), (40.5) and (40.7), and by inserting the numerical values of c, ϵ_0 and π^2 we have, for the cylindrical shape,

$$\xi = \frac{V'_R{}^2}{2\omega W_F} = 4.9 \times 10^2 a \qquad .. \quad (40.8)$$

Substituting from (40.3) and (40.4) in (40.2) leads to Q also being expressed in terms of a; then we have

$$\zeta_{10} = \xi Q_{10} = \frac{1.29 \times 10^8 \lambda_0^{\frac{1}{2}} a^2}{2.61a + 1} \qquad (40.9)$$

The ratio a is plotted in Fig. 3 as a function of the characteristic voltage. For $V_{OB} =$ 5,000 volts, $a = 3.5 \times 10^{-2}$, and

$$\frac{\zeta}{\lambda^{\frac{1}{2}}} = 1.43 \times 10^5 \qquad .. \quad .. \quad (40.10)$$

and

$$Q_{10} = 2{,}650 \qquad .. \quad .. \quad (40.11)$$

Substituting $a = 3.5 \times 10^{-2}$ in (40.8) because $V_{OB} = 5{,}000$ volts, we have

$$\xi = 17.2 \qquad .. \quad .. \quad (40.12)$$

The effective cross-sectional area of the beam may, perhaps, reasonably be taken as that of a disc of a radius equal to about $r_0/5$.

Then

$$A_0 = \frac{\pi r_0{}^2}{2.5 \times 10} = 1.26 \times 10^{-1} r_0{}^2$$

and from (40.3),

Therefore

$$A_0/\lambda_0{}^2 = 1.84 \times 10^{-2} \; .. \quad .. \quad (40.13)$$

For the present purpose, let $\Delta =$ unity — a rather optimistic figure (see Fig. 4).

Collecting all these results for a reference wavelength λ_0 of 0.1 metre, and $V_{OB} = 5{,}000$ volts, we have, (from (37)),

$$\Psi_{10} = \frac{\xi Q_{10}}{R_{B10}} = 8.3 \qquad .. \quad .. \quad (40.14)$$

which, (by reference to Figs. 6 and 7) is quite useless.

This example has been shown in detail for purposes of explanation. Alternately, of course, ζ_{10} can be computed in the form of equation (35).

11.2. *Rectangular-box cavity oscillating at the lowest frequency (H_{011} mode).*

A similar set of calculations to those for a cylinder may be made for the cavity illustrated in Fig. 9 on the assumption that it is oscillating in the H_{011} mode. The beam path must then be in the y-direction and in the centre of the box, because this is the path and direction of maximum electric field for the mode in question. Therefore ζ will be as great as is possible with this resonator and this mode.

For $V_{OB} = 5{,}000$ volts, then, it will be found that

$$\zeta/\lambda^{\frac{1}{2}} = 1.5 \times 10^5 \qquad .. \quad .. \quad (41.1)$$

and

$$\xi = 17.7 \qquad .. \quad .. \quad (41.2)$$

At a reference wavelength of 0.1 metre,

$$Q_{10} = 2680 \qquad .. \quad .. \quad (41.3)$$

and

$$\zeta_{10} = Q_{10}\xi = 4.7 \times 10^4 \; .. \quad .. \quad (41.4)$$

These values are very nearly the same as for the cylinder.

Let, as before, $\Delta =$ unity, and

$$\frac{A_0}{\lambda^2} = 1.84 \times 10^{-2} \qquad .. \quad .. \quad (41.5)$$

Then, for $V_{OB} = 5{,}000$, and $\lambda_0 = 0.1$ metre,

$$\Psi_{10} = \frac{\zeta_{10}}{R_{B10}} = 8.72. \qquad .. \quad .. \quad (41.6)$$

11.3. *The low efficiencies of simple shapes of cavity.*

From, for instance, equation (40.2) the Q of the cylinder at the E_{010} mode increases asymptotically with z_0. At $\lambda_0 = 0.1$ m it is about 2,660. Also, from (40.7) ξ increases directly with z_0; but both these parameters are limited by the transit-angle considerations which determine a. The value of ξ, from (40.12) and (41.2) is only about 17 (for $V_{OB} = 5{,}000V$) in the case of both the simple shapes which have been computed. It does not, of course, vary with wavelength.

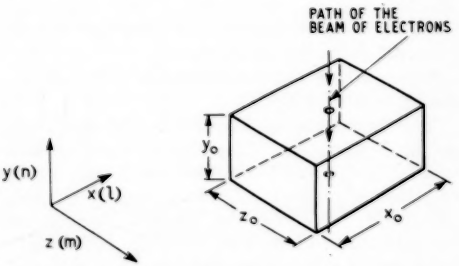

Fig. 9. *An electron-beam path used with a rectangular-box resonator. The mathematical suffixes and coordinates are shown. The load is not shown.*

The above results show that the limit set by ϕ_0 to that dimension of the resonator which lies in the direction of the beam path (the z_0 direction in the case of the cylinder, and the y-direction in the case of the rectangular box) puts a limit to both Q_0 and

to the voltage parameter ξ, and therefore severely limits $\zeta_0 = \xi Q_0$.

The general conclusion from the two preceding examples is that those simple resonator shapes which have low enough characteristic voltages to be useful are capable of only a very limited performance at the shorter microwavelengths.

11.4. *Distorted cavities.*

It was early realized (Bib. 8) during work on resonator-valves, that improved results might be expected if the sides of the resonator

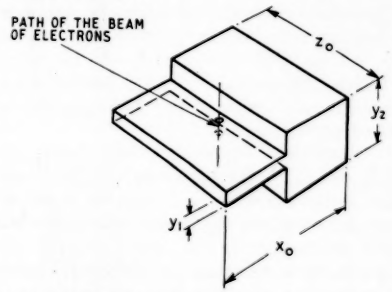

Fig. 10. *A modified rectangular-box resonator and electron-beam path.*

were "pinched in" opposite the ends of the path of the beam. A concentration of electric field would then occur along the path and was expected to cause the resonator to "match" the beam characteristics.

Unfortunately, the analytical treatment of such "distorted" shapes is usually difficult and time-consuming, and the results when obtained not always very informative. In fact, an empirical technique of study by means of oversize models (to be described later) seems, so far, to be much more informative than any analytical attack.

11.5. *A "stepped" rectangular-box resonator.*

The "distorted" rectangular-box type of resonant cavity illustrated in Fig. 10 has almost the same resonant wavelength as a plain rectangular box (Fig. 9) of the same dimensions $x_0 z_0$ when excited in the corresponding H_{011} mode with the electric field in the y direction.

With $\frac{y_1}{y_2} = \frac{1}{9}$, $V_{0B} = 5{,}000$V, and with the beam injected across the narrower dimension y_1, as shown, an approximate calculation yields

$$\frac{\zeta}{\lambda^{\frac{1}{2}}} = 2.66 \times 10^5 \quad .. \quad .. \quad (42.1)$$

$\xi = 17$, as in the rectangular box; but Q is increased by 1.75 times. ζ_{10} is therefore 1.75 times better than for the plain rectangular box; but A_0 is rather smaller due to the field distortion near the "step." Ψ_{10} is not raised to more than 14 or so by this use of a "step" in a rectangular cavity. Thus the improvement is not a useful one.

One may note that, on wavelengths longer than the lower end of the microwave band, much less stringent requirements hold, and it is significant that early velocity-modulated tubes were successful when they were tried out on longer waves. The difficulties increase enormously with even a small decrease of wavelength (Fig. 6.).

11.6. *A "distorted" cylindrical resonator.*

Any successful attempt to make a useful advance in resonator performance requires more information than is economically obtainable by analytical means—if indeed one can obtain it at all in that way in the present state of the art—but it will be well worth while to take advantage of some ingenious mathematical evaluations of field integrals in certain "distorted" cavities which have beer published by Hansen & Richtmeyer (Bib. 7). These particular results refer to the lowest-frequency mode of oscillation only.

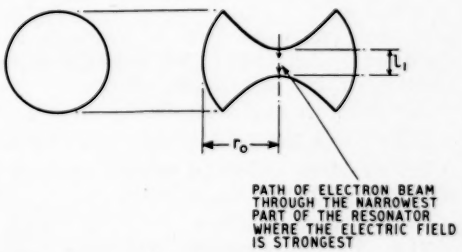

Fig. 11. *A "distorted" cylindrical resonator and electron-beam path.*

These investigators have solved an integral

$$R = 4\pi Q \frac{\left[\int \mathbf{E}\, d\mathbf{s}\right]^2}{\frac{2\pi}{\lambda}\int \mathbf{E}^2\, d\mathbf{v}}\, c \text{ (e.m.u.)}$$

[see their equation (2)] for certain resonator shapes, including those for an "hour-glass" distorted-cylindrical configuration (Fig. 11). It may be shown that, if the path of integration $\mathbf{s}$ in the above equation be chosen so that the numerator is equal to the quantity

$V'_R{}^2 \doteq (V_{OB}M_0)^2$ of the present paper, then $R = \zeta$; and one of Richtmeyer and Hansen's cavity shapes may be directly examined in the light of the present analysis.

In Fig. 11, the path of the electron beam is as shown. This path lies in the direction of the electric field and is positioned where this is greatest. The distance between the two inner surfaces of the resonator along this path is taken to be the distance l_1 of the present analysis. It is clear from Hansen & Richtmeyer's work, that the shapes of cavity corresponding to values of their parameter σ_0 greater than about 10 correspond to values of V'_{OB} decreasing from about 140 over the useful range. Moreover ζ varies little with V'_{OB} over this range. (This latter is an important property of some kinds of resonators).

Thus, for $V_{OB} = 5{,}000$V, it is found that, for the cavity shape of Fig. 11, we have

$$\zeta/\lambda^{\frac{1}{2}} = 3.9 \times 10^6 \quad .. \quad .. \quad (42.1)$$

which is more than a ten times improvement over the simple cylinder, or rectangular box.

This particular shape of resonator is suitable only for "focused" beams of electrons. A value of A_0—which will depend, with Δ, partly upon non-analytical factors—may be chosen.

We have

$$\lambda_0 = 3.7r_0 \quad .. \quad .. \quad .. \quad (42.2)$$

and, from (40.3) the area A_0 available for the beam is, therefore, rather less at a given wavelength than in a simple cylinder.

Therefore, we may state that

$$A_0/\lambda^2 = 9.2 \times 10^{-3} \quad .. \quad .. \quad (42.3)$$

Taking Δ as unity as before, we have, for $\lambda_0 = 10$ cm. and $V_{OB} = 5{,}000$,

$$\Psi_{10} = \zeta_{10}/R_{B10} = 114 \quad .. \quad .. \quad (42.4)$$

The voltage parameter has also risen (for $V_{OB} = 5{,}000$V) to

$$\xi = 90 \quad .. \quad .. \quad (42.5)$$

as compared with about 17 for the simple cylindrical and rectangular shapes of cavity.

Furthermore

$$Q_{10} = 13{,}600 \quad .. \quad .. \quad (42.6)$$

as compared with about 2,600 for the simple shapes. Even in the case of the "stepped" rectangular-box resonator Q_{10} equals only 4,500. It is noteworthy that the improvement found in the "hour-glass" resonator is not only in respect of ξ, but in respect of Q_{10} as well, and, in both cases, the increase is about 5 times.

Referring back to Figs. 6 and 7, it will be observed from (42.4) that the "hour-glass" shape is quite an efficient and useful cavity for use over a large part of the shorter microwave band such that $\lambda_0 = 0.1$ m.

Hansen & Richtmeyer also calculated R for another shape consisting of a sphere with two re-entrant cone-shaped "pole pieces" between the tips of which the beam of electrons is intended to travel. Nearly twice the value of the R factor is then obtained compared with the shape of Fig. 11; but it is difficult to correlate these results with the present analysis, as there is apparently no assumption made in this part of their work as to the distance l_1 or beam area A_0. Nevertheless the results are of interest as apparently indicating the same general level of performance.

It may therefore be concluded that "distorted" cavities are those which promise by far the best results, and that, when the minimum possible operative wavelength is required, the resonator shape is of primary importance. These examples also provide a guide to the orders of Q_0 and ξ which are necessary.

It will be of interest to note qualitatively that the "best" resonator, for a given A_0, will, from the right-hand side of (35), be that which has the greatest concentration of electric field along the beam path, and the least density of the magnetic field elsewhere: that is, one having a shape so that it has a small area of "electric antinode" as nearly as possible confined to the beam path itself; and a large area of copper surface elsewhere so as to keep the copper loss down. Cylindrical, and rectangular box resonators are all "bad" by this criterion, and distorted shapes with pole pieces are "good."

The resonator shape must be chosen with regard to the desired value of the characteristic voltage V_{OB}.

11.7. *Field shapes for combined beam modulation and power transfer.*

In the foregoing analysis it has been tacitly assumed that the beam is modulated by means external to the system analysed.

In fact, however, a part of the field in the resonator may be arranged to modulate the beam, either by varying its velocity (before the electrons pass into a "drift-space") or by transversely deflecting it.

(To be concluded)

LINEARITY RANGE OF NOISE-MEASURING AMPLIFIERS*

By R. L. Bell, B.Sc.

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IN making quantitative measurements on electronic noise phenomena, such as thermal agitation, shot effect, etc., the use of an amplifier having a finite signal-handling capacity necessitates the investigation of the relation between the noise input and output voltages for such an amplifier. This arises from the fact that there is a finite probability of the occurrence of any peak value, however large, of a noise voltage, and all peaks greater than a certain value will produce overloading (i.e., non-linearity) of the amplifier.

The object of this note is to examine the error in mean square and root-mean-square measurement resulting from an assumption of linear amplification in two cases:

(a) an amplifier with a resistive output load,

(b) an amplifier with a selective output load.

It is well known that the instantaneous value v of such a noise voltage has a normal probability distribution in time, with mean square σ^2 and mean zero, all values between $-\infty$ and $+\infty$ being possible:

$$\frac{dp(v)}{dv} = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{v^2}{2\sigma^2}\right) \quad .. \quad (1)$$

The expression on the right of (1), however, tends rapidly to zero for

$$|v| > 4\sigma \quad .. \quad .. \quad .. \quad .. \quad (2)$$

so that voltages greater than 4σ are of rare occurrence.

The usual practice is either to measure the mean square directly on some square-law device, the mean square being given by

$$\sigma^2 = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} v^2 \exp\left(-\frac{v^2}{2\sigma^2}\right) dv \quad .. \quad (3)$$

or to estimate the root-mean-square by rectifying and measuring the mean rectified value:

$$\frac{\sigma}{\sqrt{2\pi}} = \frac{1}{\sigma\sqrt{2\pi}} \int_{0}^{\infty} v \exp\left(-\frac{v^2}{2\sigma^2}\right) dv \quad (4)$$

* MS. accepted by the Editor, July 1946.

Modifications of either method are of course used.

Load. Case (a).

We can take the noise-measuring apparatus as having a "limiter type" characteristic:

$$\begin{aligned} v_2 &= av_1 \quad \text{for} \quad 0 \leqslant v_1 \leqslant B \\ &= aB \qquad\quad v_1 \geqslant B \\ &= 0 \qquad\quad v_1 \leqslant 0 \end{aligned} \quad .. \quad .. \quad (5)$$

where v_1 and v_2 are the instantaneous values of input and output voltage respectively, and a and B are suitable circuit constants.

The proportion of time that v_2 spends in the range av_1, $a(v_1 + dv_1)$, is, from (1),

$$\frac{dv_1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{v_1^2}{2\sigma^2}\right) \quad 0 \leqslant v_1 \leqslant B \quad .. \quad (6)$$

and the proportion of time that $v_2 = aB$ is

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{B}^{\infty} \exp\left(-\frac{v_1^2}{2\sigma^2}\right) dv_1 \quad .. \quad .. \quad (7)$$

which is the probability that $v_1 \geqslant B$.

Hence the mean output voltage:

$$\overline{v_2} = \frac{a}{\sigma\sqrt{2\pi}} \int_{0}^{B} v_1 \exp\left(-\frac{v_1^2}{2\sigma^2}\right) dv_1 + \frac{aB}{\sigma\sqrt{2\pi}} \int_{B}^{\infty} \exp\left(-\frac{v_1^2}{2\sigma^2}\right) dv_1$$

i.e.,

$$\frac{\overline{v_2}}{aB} = \frac{\sigma}{B\sqrt{2\pi}} \left[1 - \exp\left(-\frac{B^2}{2\sigma^2}\right)\right] + \operatorname{erfc}\left(\frac{B}{\sigma}\right) \quad (8)$$

where

$$\operatorname{erfc}(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp(-y^2/2)\, dy \quad (9)$$

and is the tabulated complementary error function.

Similarly the mean-square output voltage:

$$\overline{v_2^2} = \frac{a^2}{\sigma\sqrt{2\pi}} \int_{0}^{B} v_1^2 \exp\left(\frac{v_1^2}{2\sigma^2}\right) dv_1 + \frac{a^2B^2}{\sigma\sqrt{2\pi}} \int_{B}^{\infty} \exp\left(-\frac{v_1^2}{2\sigma^2}\right) dv_1$$

that is, using (9)

$$\frac{\overline{v_2^2}}{a^2B^2} = \frac{1}{2}\frac{\sigma^2}{B^2} - \frac{\sigma}{B\sqrt{2\pi}}\exp\left(-\frac{B^2}{2\sigma^2}\right) + \left(1 - \frac{\sigma^2}{B^2}\right)\mathrm{erfc}\left(\frac{B}{\sigma}\right) \quad .. \quad (10)$$

Selective Load. Case (b)

We shall treat such a selective amplifier as a device which produces a current with positive and negative peaks limited (i.e., "chopped off" symmetrically at a predetermined level) when a sufficiently large alternating voltage is applied to the input;

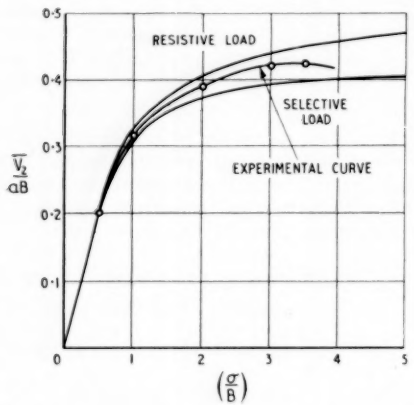


Fig. 1. *Mean output voltage versus r.m.s. noise input voltage σ.*

the output load then picks out the fundamental component of this current. If in addition the actual load responds to harmonics, the behaviour of the circuit will be intermediate between that of the previous case and this. A random noise voltage or current can be expressed in the form of an amplitude- and phase-modulated sine-wave:

$$v(t) = P(t)\sin[\omega t - \phi(t)] \quad .. \quad (11)$$

where P is distributed between 0 and ∞ with a density

$$\frac{dp(P)}{dP} = \frac{P}{\sigma^2}\exp\left(-\frac{P^2}{2\sigma^2}\right) \quad .. \quad .. \quad (12)$$

in which σ^2 is the mean square of v and, in the case where the noise is confined to a narrow frequency band about the value $\frac{\omega}{2\pi}$, P has a definite physical significance as the envelope of the noise. The distributions (1) and (12) are deducible from each other.

A little consideration shows that for the purposes of the problem, the amplifier can be supposed to achieve its selectivity in the stages preceding the output stage, this stage itself having a load impedance which is of constant magnitude over the pass band and zero outside it.

We have for the output current i, abbreviating $P(t)$ to P, etc., and introducing a suitable circuit parameter g,

$$\begin{aligned} i &= gP\sin[\omega t - \phi] & |v_1| &\leqslant B \\ &= \pm gB & |v_1| &\geqslant B \end{aligned} \quad (13)$$

The amplitude of the fundamental component of this is

$$\begin{aligned} A_1 &= \frac{2g}{\pi}\left[P\sin^{-1}\frac{B}{P} + \frac{B}{P}\sqrt{P^2 - B^2}\right] & P &\geqslant B \\ &= gP & P &\leqslant B \end{aligned} \quad (14)$$

and, for the instantaneous value of the output voltage (α being some phase angle determined by ϕ and the circuit),

$$v_2 = A_1 Z\sin(\omega t - \alpha)$$

where Z is the magnitude of the load impedance.

In keeping with our previous notation,

$$\overline{v_2} = \frac{Z\overline{A}_1}{\pi}$$

and

$$\overline{v_2^2} = \tfrac{1}{4}Z^2\overline{A}_1^2$$

being parameters of the half-wave only.

Whence

$$\frac{\pi\overline{v_2}}{Z} = \frac{1}{\sigma^2}\int_0^B gP^2\exp\left(-\frac{P^2}{2\sigma^2}\right)dP + \frac{1}{\sigma^2}\int_B^\infty \frac{2g}{\pi}\left[P\sin^{-1}\frac{B}{P} + \frac{B}{P}\sqrt{P^2 - B^2}\right]P\exp\left(-\frac{P^2}{2\sigma^2}\right)dP$$

which reduces to:

$$\frac{\pi\overline{v_2}}{gZB} = \frac{1}{B}\int_0^B \exp\left(-\frac{P^2}{2\sigma^2}\right)dP + \frac{2}{B\pi}\int_B^\infty \left[\sin^{-1}\frac{B}{P} - \frac{B}{P^2}\sqrt{P^2 - B^2}\right]\exp\left(-\frac{P^2}{2\sigma^2}\right)dP \quad (15)$$

Also, for the mean square,

$$\frac{4\overline{v_2^2}}{Z^2} = \frac{g^2}{\sigma^2}\int_0^B P^3\exp\left(-\frac{P^2}{2\sigma^2}\right)dP + \frac{4g^2}{\pi^2\sigma^2}\int_B^\infty \left[P\sin^{-1}\frac{B}{P} + \frac{B}{P}\sqrt{P^2 - B^2}\right]^2 P\exp\left(-\frac{P^2}{2\sigma^2}\right)dP \quad (16)$$

Unfortunately, the expressions on the right of (15) and (16) do not appear to be integrable in terms of known functions. Substituting $P = \sigma x$ in the first, and $Py = B$ in the second pair of integrals of (15) and (16) respectively; and writing the gain $gZ = a$, we have

$$\frac{\overline{v_2}}{aB} = \frac{\sigma}{\pi B}\int_0^{\frac{B}{\sigma}} \exp\left(-\frac{x^2}{2}\right)dx + \frac{2}{\pi^2}\int_0^1 \exp\left(-\frac{B^2}{2\sigma^2y^2}\right)\left[\sin^{-1}y - y\sqrt{1-y^2}\right]\frac{dy}{y^2} = \frac{\sigma}{B}\sqrt{\frac{2}{\pi}}\,\mathrm{erf}\left(\frac{B}{\sigma}\right) + \frac{2}{\pi^2}\int_0^1 \exp\left(-\frac{B^2}{2\sigma^2y^2}\right)\left[\frac{\sin^{-1}y}{y} - \sqrt{1-y^2}\right]dy \quad \ldots \quad (17)$$

and

$$\frac{\overline{v_2^2}}{a^2B^2} = \tfrac{1}{2}\frac{\sigma^2}{B^2} - \left[\exp\left(-\frac{B^2}{2\sigma^2}\right)\right]\left[\tfrac{1}{4} + \tfrac{1}{2}\frac{\sigma^2}{B^2}\right] + \frac{B^2}{\pi^2\sigma^2}\int_0^1 \left[\frac{\sin^{-1}y}{y} + \sqrt{1-y^2}\right]^2 \times \exp\left(-\frac{B^2}{2\sigma^2y^2}\right)\frac{dy}{y^3} \quad \ldots \quad (18)$$

The integrals have been evaluated numerically for different values of $\frac{\sigma}{B}$ and equations (8), (17); (10) and (18) are shown plotted in Figs. 1 and 2 respectively. Experimental points of "mean half" (mean rectified) output voltage versus r.m.s. noise input for an amplifier with a broadly-tuned output load are shown superposed on the curves of Fig. (1).

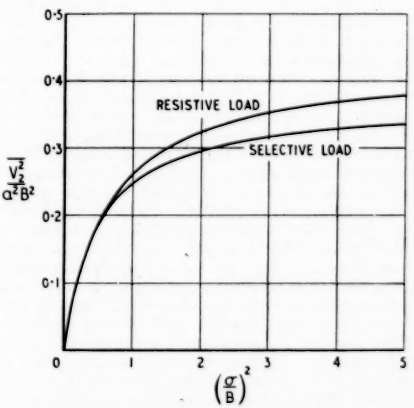


Fig. 2. *Mean-square output voltage versus mean-square-noise input voltage σ^2.*

In both cases (a) and (b) we have for the mean-rectified output voltage, as $\sigma/B \to 0$,

$$\overline{v_2} \to \frac{a\sigma}{\sqrt{2\pi}} \quad \ldots \quad (19)$$

and for the mean-square output voltage,

$$\overline{v_2^2} \to \tfrac{1}{2}a^2\sigma^2 \quad \ldots \quad (20)$$

which are clearly true from elementary considerations. We may now examine the error involved in assuming that (19) and (20) are actual equalities when σ/B is small.

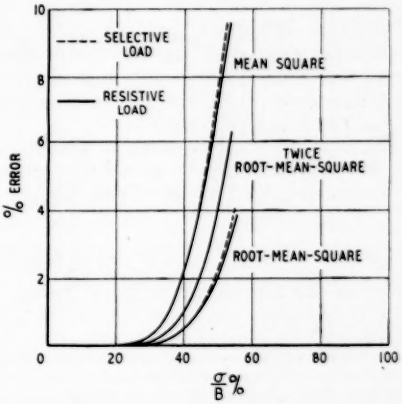


Fig. 3. *Variation of percentage measurement error with r.m.s. noise input voltage σ.*

For a resistive load, we have, from (8) and (10),

$$\frac{\frac{a\sigma}{\sqrt{2\pi}} - \overline{v_2}}{\frac{a\sigma}{\sqrt{2\pi}}} = \exp\left(-\frac{B^2}{2\sigma^2}\right) - \sqrt{2\pi}\,\frac{B}{\sigma}\,\mathrm{erfc}\left(\frac{B}{\sigma}\right) \quad \ldots \quad (21)$$

$$\frac{\tfrac{1}{2}a^2\sigma^2 - \overline{v_2^2}}{\tfrac{1}{2}a^2\sigma^2} = \sqrt{\frac{2}{\pi}}\,\frac{B}{\sigma}\exp\left(-\frac{B^2}{2\sigma^2}\right) + 2\left(1 - \frac{B^2}{2^2}\right)\mathrm{erfc}\left(\frac{B}{\sigma}\right) \quad \ldots \quad (22)$$

and from (17) and (18), for a selective load,

$$\frac{\frac{a\sigma}{\sqrt{2\pi}} - \overline{v_2}}{\frac{a\sigma}{\sqrt{2\pi}}} = 2\,\mathrm{erfc}\left(\frac{B}{\sigma}\right) - \frac{2B}{\pi\sigma}\sqrt{\frac{2}{\pi}}\int_0^1 \left[\frac{\sin^{-1}y}{y} - \sqrt{1-y^2}\right]\times \frac{1}{y}\exp\left(-\frac{B^2}{2\sigma^2y^2}\right)dy \quad \ldots \quad (23)$$

and

$$\frac{\frac{1}{2}a^2\sigma^2 - \overline{v_2}^2}{\frac{1}{2}a^2\sigma^2} = \left[1 + \frac{B^2}{2\sigma^2}\right]\exp\left(-\frac{B^2}{2\sigma^2}\right) - \frac{2B^4}{\pi^2\sigma^4}\int_0^1\left[\frac{\sin^{-1}y}{y} + \sqrt{1-y^2}\right]^2 \times \frac{1}{y^3}\exp\left(-\frac{B^2}{2\sigma^2y^2}\right)dy \quad .. \quad (24)$$

The errors given by equations (21) to (24) are shown plotted as functions of $\frac{\sigma}{B}$ in Fig. 3. It is to be observed that, in either case, for a 1 per cent error in the mean square, σ may be as high as 35 per cent of B, and for a similar error in the root-mean-square, σ must rise to 43 per cent of B.

The equivalent pass range at the output, aB, may quite easily be made of the order of several hundred volts, depending on the h.t. supply, so that these percentages infer quite a high upper limit to the permissible output voltage, $a\sigma$.

A similar rough rule was given some years ago* limiting the maximum value of σ in the case of mean square measurement to 10 per cent of B, and, so far as the author is aware, nothing has since been added to this. It appears, however, from Fig. 3, that much greater voltages than this can in fact be handled without appreciable error.

It is also clear from Fig. 3 that the fractional error in the mean square is always more than twice that in the root-mean-square, so that, with conventional amplifiers, it is inherently more accurate to measure root-mean-square values of noise voltages.

* E. B. Moullin: "Spontaneous Fluctuations of Voltage," Oxford, 1938. P. 218.

BOOK REVIEWS

Antennae—An Introduction to Their Theory.

By J. Aharoni. Pp. 265 + viii, with 149 Figs. Oxford University Press. Price 25s.

The object of this book, according to the preface, is to give a comprehensive account of the progress that has been achieved in recent years by applied mathematicians in calculating the properties of antennae. Although the word "introduction" occurs in the title, it is not a book for the beginner; it assumes that the reader has considerable mathematical knowledge, especially of vector algebra. The first section of 86 pages is entitled "Antennae and boundary-value problems" and deals with Maxwell's equations, the Hertzian vector and retarded potentials, the fields of infinitely long wires and coaxial cones, and the free and forced oscillations of spheres and spheroids. The second and main section of 140 pages is entitled "Antennae and integral equations," and deals with magnetic and electric dipoles, coupled Hertzian dipoles, straight thin-wire antennae, the receiving properties of a straight antenna, polar diagrams of broadside arrays, earth effects, and direction finding. The final section treats the antenna as a type of waveguide in the manner developed by Schelkunoff. The treatment throughout is based on the work done in recent years by Hallèn, Ryder, R. King, C. W. Harrison, M. C. Gray and Schelkunoff, which is all discussed.

In the introduction on p. 1, it is stated that "the value of the current at a generator is of particular importance as the ratio of the e.m.f. supplied by the generator to this current defines the impedance which the aerial offers to the generator and, for maximum power, this impedance must be matched to the internal impedance of the generator." If a generator has an e.m.f. of 100 and an internal impedance of $10 + j10$, what impedance must the external load have in order that maximum power may be supplied to it? If it be made $10 - j10$ the current is $100/20 = 5$ and the power $P = I^2R = 250$. If it be made $10\sqrt{2} + j0$ the current is 3.83 and $P = 207$. If it be made $10 + j0$ the current is 4.47 and $P = 200$. Finally, if it be made $10 + j10$, i.e., an exact match, the current is 3.53 and $P = 125$. One should evidently be rather careful when talking about matching impedances. The maximum output is obtained when the external impedance is the conjugate of the internal impedance. Something seems to have gone very wrong on p. 5, where Fig. 1 shows a rod of dielectric material of dielectric constant ϵ_2, and it is described in the text as a thick wire of dielectric constant ϵ_2, but it suddenly becomes a current-carrying conductor of conductivity σ, which it was evidently intended to be from the beginning.

Symbols are always a trouble in a book of this kind. The author cannot be expected to be familiar with the activities of the British Standards Institution. Instead of κ he still uses ϵ for the dielectric constant, as so many authors do, and it has this in its favour that, as a Greek initial, it is the electric counterpart of the magnetic μ. We were surprised to find e instead of q as the symbol for quantity or charge; small letters and capitals seem to be used indiscriminately and so we have $I = de/dt$, which is intended to convey the information that the instantaneous value of the current is equal to the rate of change of the quantity or charge. The author could probably reply that the symbols are those employed in some of the various papers of which the book is largely a résumé and discussion. Anyone who is meticulously careful in the use of British Standard symbols will find it difficult to realise that $e\,e^{-\alpha t}/q$ represents a charge dying away exponentially divided by a cross-sectional area (German *Querschnitt*). These remarks must not be regarded as an adverse criticism of this book in particular, but rather as a comment on the symbolic difficulties that are met with when electrical engineers turn to books written by mathematical physicists. The book is well produced; the printing, illustrations, paper and binding are all excellent.

G. W. O. H.

Drafting for Electronics

By L. F. B. CARINI. Pp. 211+ix. Published by McGraw-Hill Publishing Co. Ltd., Aldwych House, London, W.C.2. Price 12s. 6d.

This book is intended to help draughtsmen engaged in the production of circuit diagrams of electronic apparatus. It opens by discussing the kind of drawing-office equipment which is needed and gives a great deal of information about how the various instruments should be used. The steps in the drawing of various circuit symbols and lettering are indicated.

All this occupies roughly one-third of the book and it forms an excellent introduction to drawing which should be of great assistance to the beginner. The author's style is clear and as long as he confines himself to drawing one can have little but praise for his work. It is true that he is somewhat inconsistent. After deprecating the use of open arrow heads he uses them more frequently than not in his diagrams. Then, too, there is an actual error in one of his circuit diagrams, Fig. 81.

When the author comes to deal with the layout of a circuit diagram the book becomes much less satisfactory. His main recommendation is that the circuit symbols should be uniformly spaced, but this is just what they should not be. The aim of the diagram is to show the details of a circuit in such a way that the reader can follow them most easily. This is achieved, not by a uniform spacing of the symbols, but by their careful grouping.

The bad effect of the uniform spacing which the author recommends is most convincingly demonstrated by an example from his own book. In Fig. 115 he gives a rough engineering sketch of a circuit and in Fig. 116 the draughtsman's finished version of it. The rough sketch is incomparably the better of the two, simply because it is better spaced. The two are on facing pages and when one looks for any circuit detail one finds that one's eyes automatically go to the rough sketch. This sketch is by no means perfectly spaced, but it is better spaced than the finished version

The author deprecates the practice of drawing diagrams so that the various supply bus-bars are placed in order of their potential with respect to earth. This is common British practice but not American and this has perhaps coloured the author's views. The practice is one to be commended, not deprecated, however, for it does greatly assist the user of the diagram. The author says that the draughtsman is not paid to be an electrical engineer and that it is not his job to sort out the bus-bars in order of potential. But it is his job to produce a diagram which gives to the *user* a maximum of simplicity and legibility. This is all that matters; and in fact, intelligent co-operation between engineer and draughtsman is all that is needed.

The author has omitted at least one very important " Don't." It is, never on any account draw more than three adjacent and equally spaced bus-bars. It is just impossible to follow more without using a pencil to trace them out. His Fig. 90 shows five such lines and although the practice is not uncommon, it is bad practice.

So far as the purely drawing side is concerned the book is excellent, but the advice on circuit diagram layout and arrangement cannot be considered as anything but representative of a bad practice which one hoped was a thing of the past. The book concludes with an extremely useful list of visual aids to drawing, mainly film and film strips.

W. T. C.

Electrical Transmission in Steady State

By PAUL J. SELGIN. Pp. 427 + vii, with 99 Figs. Published by McGraw-Hill Publishing Co. Ltd., Aldwych House, London, W.C.2. Price 25s.

The author was formerly an instructor at the Polytechnic Institute of Brooklyn and the book is based on a series of lectures given during the war under the War Training Programme. It is mainly devoted to the steady-state transmission along lines, but the last four of the seventeen chapters deal with inductive coupling and transformers, capacitive coupling, four-pole theory applied to the vacuum tube, and h.f. amplifiers. As the title indicates, transient phenomena are excluded. As the author says in the preface, " a book such as this must be built upon a solid mathematical framework and cannot be 'light reading.' Yet it must remain accessible to readers of average mathematical preparation." The treatment throughout is very thorough, illustrated by many diagrams and examples. The first chapter deals with a four-terminal network as a circuit element, and in laying foundations the author discusses the mechanical-electrical analogies and the principles of duality and superposition. One is used to seeing the Helmholtz " make and break " theorem referred to as Thévenin's theorem, but it was new to us to see the dual equivalent with the constant-current generator and the parallel admittance referred to as " Norton's equivalent." The author gives no reference or indication why he gives this name to this equivalent circuit. In *Wireless Engineer* of August, 1943, A. Bloch gave an interesting account of its history and suggested that it might be called the Helmholtz-Russell theorem.

Following chapters discuss the application of network theory to transmission lines, distortion, reflection, the use of mapping methods, the use of lines as matching devices, stubs, etc., all treated very fully and clearly. A brief review is then given of electromagnetic theory and the application of Maxwell's equations to circuit elements and exponential lines. There seems to be some confusion in the author's mind on the question of distortion for on p. 80 he says, " If the attenuation varies with frequency, the mutual proportion of the components of a sound will be affected by transmission, giving rise to *amplitude distortion*. This is sometimes called *frequency distortion*." The italics are in the original, thus emphasizing the muddle. A student who had used this book would be rather puzzled if asked in an examination to distinguish between amplitude and frequency distortion. We might refer the author to pages 91 and 92 of Terman's Fundamentals of Radio where he will find the terms explained. There is a printer's error on p. 276, where the velocity of the wave in air is given as 3×10^3 m/sec; also we note that capacitance is sometimes 0.1 μF, sometimes 0.1 μf and sometimes merely 0.1 μ, but these are minor details and detract little from the value of the book, which we can recommend to anyone interested in transmission problems. It is very well printed and very clearly illustrated and utilizes the transmission problems to give a sound grounding in electromagnetism.

G. W. O. H.

CORRESPONDENCE

Letters of technical interest are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Valve Equivalent Circuit

To the Editor, "Wireless Engineer."

SIR,—During the past fifteen years, use of the constant-current generator circuit as an alternative first-order equivalent representation for the vacuum tube* has become widespread. It does not appear, however, to be generally recognized that this representation is only a limited equivalent to the more fundamental concept of a constant-voltage generator circuit to represent first-order operation of the vacuum tube. I have not made a diligent search of the literature to determine whether the limited nature of the equivalent circuit has been pointed out before. It may be that it has, but it would appear that the matter is worth emphasizing. The limited nature of the equivalence may be illustrated in the following way. Fig. 1 shows the well-known constant-voltage generator circuit Assuming a harmonic variational grid voltage of r.m.s. value E, the power developed in the external load resistance is

$$P = \frac{\mu^2 E^2}{(r_p + R)^2} \cdot R \qquad (1)$$

and the power developed in the internal resistance is

$$P' = \frac{\mu^2 E^2}{(r_p + R)^2} \cdot r_p \qquad (2)$$

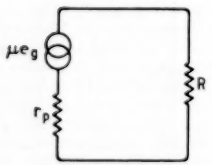


Fig. 1. *The Constant-Voltage Generator Circuit.*

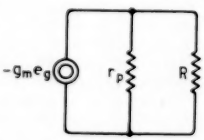


Fig. 2. *The Constant-Current Generator Circuit.*

Fig. 2 shows the now equally well-known constant-current generator circuit. For this arrangement the power developed in the external load resistance is

$$P = \frac{\mu^2 E^2}{(r_p + R)^2} \cdot R \qquad (3)$$

and the power developed in the internal resistance is

$$P' = \frac{\mu^2 E^2}{(r_p + R)^2} \cdot \frac{R^2}{r_p} \qquad (4)$$

On comparing (1) and (3) it will be observed that the two circuit representations give the same results for the power developed in the external load resistance. On comparing (2) and (4) it will be observed that the two circuit representations do not give identical results for the power developed in the internal resistance. Is (2) correct, or is (4) correct, and why? The answer to this question is that (2) is correct, the reason being that Fig. 2 is equivalent to Fig. 1 only for the current through, and voltage across, the external load resistance. Fig. 2 is not equivalent to Fig. 1 as regards the current through, and voltage across, the internal load resistance.

Once stated, these remarks would appear to be obvious, but it may not be amiss to provide a proof, particularly since the proof will serve to emphasize the fundamental nature of the circuit shown in Fig. 1.

The anode current of the vacuum tube is a function of both grid and anode voltages. Assume that each of these voltages consists of a steady polarizing component and a relatively small variable component. The functional relationship may then be expanded in a double Taylor's series about the steady voltages, getting to a first-order approximation,

$$I_p = I_p(E_g, E_p) + e_g \cdot \frac{\partial I_p}{\partial E_g} + e_p \cdot \frac{\partial I_p}{\partial E_p} \qquad (5)$$

The first term of the right-hand side is the steady anode current, so that the variation in anode current is given by the remaining terms.

$$i_p = g_m \cdot e_g + g_p \cdot e_p \qquad (6)$$

where

$$g_m = \partial I_p / \partial E_g \qquad (7)$$

$$g_p = \partial I_p / \partial E_p$$

Note that

$$dI_p = g_m \cdot dE_g + g_p \cdot dE_p \qquad (8)$$

so that if $dI_p = 0$

$$-\frac{dE_p}{dE_g} = \frac{g_m}{g_p} = \mu \qquad (9)$$

Equ. (9) may be taken as a definition for μ, the amplification factor.

Now if the variation in anode voltage arises as a result of a voltage drop in an external impedance, $e_p = -i_p \cdot Z$, and from (6),

$$i_p = g_m \cdot e_g - g_p \cdot Z \cdot i_p$$

or

$$i_p = \frac{g_m \cdot e_g}{1 + g_p \cdot Z} = \frac{\mu e_g}{Z + 1/g_p} \qquad (10)$$

The physical representation of (10) is a series circuit which contains a generator of voltage μe_g, a resistance $r_p = 1/g_p$, and an impedance Z.

The voltage across the external impedance is, from (10),

$$e = -i_p \cdot Z = -\frac{g_m \cdot e_g \cdot Z}{1 + g_p \cdot Z}$$

$$= -g_m \cdot e_g \cdot \left[1/Z + g_p\right]^{-1} \qquad (11)$$

The physical representation of (11) is a shunt combination of tube conductance g_p, load admittance $Y = 1/Z$, and a constant-current generator of value $-g_m \cdot e_g$. Observe that (11) is based on the current through, and voltage across, the external impedance, as obtained from (10).

The voltage across the internal resistance may be obtained from (11) by interchanging r_p and Z. It is

$$e' = -g_m \cdot e_g \cdot \frac{r_p}{Z} \cdot \left[\frac{1}{Z} + g_p\right]^{-1} \qquad (12)$$

The physical representation of (12) is a shunt combination of tube conductance g_p, load admittance $Y = 1/Z$, and a constant-current generator of value $-g_m \cdot e_g \cdot \frac{r_p}{Z} = -\frac{\mu}{Z} \cdot e_g$. Note that as

* N. R. Bligh, "A Note on an Alternative Equivalent Circuit for the Thermionic Valve," *Experimental Wireless and the Wireless Engineer*, September 1930, Vol. 7, No. 84, pp. 480-481.

regards the internal resistance, a new value must be employed for the constant-current generator.

If $r_p = Z$ all of the circuits are equivalent, without limitation. But this is true only when the external impedance is a resistance equal to the internal resistance; it will be recognized that this represents the matched condition.

BERNARD SALZBERG.
Naval Research Laboratory,
Washington, D.C., U.S.A.

Eddy Currents in Ellipsoids

To the Editor, Wireless Engineer.

SIR,—In an article on "The Effective Impedance of a Sphere" in the December 1946 issue of *Wireless Engineer*, I said that the only finite solid object for which the eddy-current distributions caused by a varying magnetic field had been found was the sphere. I should like to say that the solution for the case of the ellipsoid of revolution has been found by M. Jouguet[1]. The solution is too complex to be of practical use except in the two special cases, (a) with the skin effect almost complete, and no internal currents and, (b) with a uniform internal field and no screening effect.

The application of these formulae to the theory of the coreless induction furnace has been very completely discussed by Jouguet in a published thesis[2] and more briefly in the other papers I have referred to.

T. S. E. THOMAS.
London S.W.11.

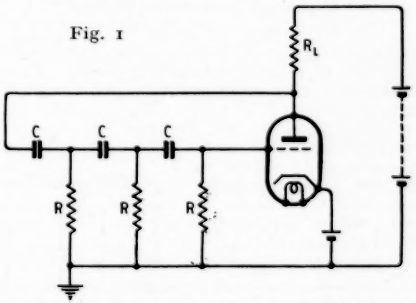
REFERENCES
[1] *Comptes Rendus*, Paris. 12th April and 31st May, 1943.
[2] M. Jouguet, "Courants de Foucault et fours à induction," Gauthier-Villars, Paris, 1944.

Phase-shift Oscillators

The Editor, The Wireless Engineer

SIR,—It has been pointed out in the literature[1] concerning RC oscillators of the type shown in Fig. 1 that if the valve is removed from its associated RC circuits, the latter become simple resistance-reactance networks, the number of meshes depending on the type of oscillator. Thence it is shown that such networks always form an aperiodically

Fig. 1



damped system; i.e., it is not possible for the differential equation for the mesh currents to have complex roots.

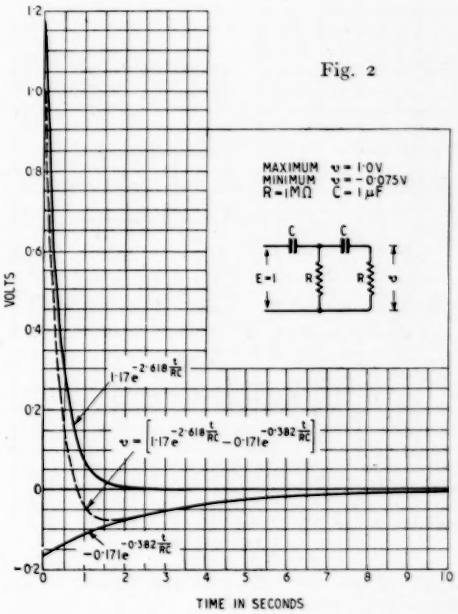
In general, for an n-mesh network it is seen that the general solution for the current is:

$$i = \sum_{1}^{n} k_l e^{-\lambda_l t}$$

where the coefficients λ_l are all real and positive; hence no oscillatory current can occur in any mesh. Therefore it is stated that the system can only oscillate if the "inherent' over-critical' damping" is balanced by a negative resistance.

It is, however, important in forming a picture of the mechanism by which oscillations build up in the

Fig. 2



phase-shift oscillator of this type (i.e., series capacitors and shunt resistors), to appreciate that the transient response to a unit voltage step of the phase-shifting network does involve voltage reversals at the output having the physical character of a heavily damped oscillation.

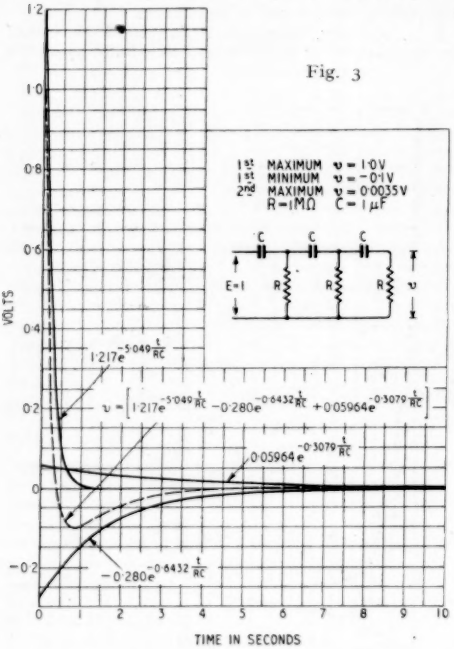
If a voltage step is applied to such a network the current in the last mesh shows a reversal, or a number of reversals, in sign. Physically, this is due to the initial charging and *subsequent discharging* of all the capacitors, except the one to which the voltage step is applied. This means that the voltage output from the network exhibits an "over-swing" effect, the number of overswings being dependent on the number of meshes present.

In particular, the transient solution of the simpler cases (e.g., 2-, 3-, and 4-mesh networks) may be evaluated using the Heaviside expansion theorem. The form of the solution for the two-and three-mesh networks is indicated in Figs. 2 and 3 respectively. It is seen that the terms in the solution of v alternate in sign, and that as t increases the numerically greatest term moves steadily along the series. There are thus one and two overswings respectively, as shown by the graphs.

In the three-mesh case the final overswing is caused by the delayed discharging of the middle capacitor due to the continued charging of the first

capacitor, after the third has discharged. This results in a small reverse charge being given to the third capacitor. It is thus this overswing property

Fig. 3



which carries the valve past the "static" point and initiates the oscillation.

It is hoped to publish subsequently a short paper dealing more fully with the mechanism of build-up in the phase shift oscillator.

Dept. of Electrical Engineering, King's College, London, W.C.2. P. G. M. DAWE. A. S. GLADWIN.

[1] Dr. E. E. Schneider, Phil. Mag. June, 1945. "A New Type of Electrical Resonance."

Degrees for Ex-Servicemen.

To the Editor "Wireless Engineer."

SIR,—There are many competent engineers who have been unable to obtain degrees, largely as a result of war service either in the Services or in industry under conditions which did not permit the necessary time for study. Many are very competent indeed and are quite capable of carrying out research or development work without technical guidance. Nevertheless, their future career would benefit greatly if they were able to obtain a degree.

The majority of these engineers are between, say, 25 and 30 years of age and have sufficient knowledge and experience to enable them to pass Inter. with very little difficulty. So far as I have been able to ascertain there is at the moment no course of study which is convenient for many of them, largely because industry is now spread throughout the country to a very much greater extent than before the war, while educational facilities are still limited to large centres of population. In addition, the general adoption of the five-day week has increased the difficulty of attending evening classes.

It is suggested, therefore, that it might be practicable to arrange courses of study at a number of technical colleges throughout the country, either in the form of a full day on Saturday or a half day on Saturday and, say, one, or at most two, evenings per week. In this respect it should be borne in mind that engineers and physicists of the grade referred to often require to visit their professional institutions for lectures and, therefore, at least one extra evening per fortnight is likely to be occupied at such gatherings.

If those engineers or physicists who are interested in such a proposal will communicate with the writer, and if the numbers prove to be sufficient, he will communicate with the Ministry of Education in order to ascertain whether something can be done to meet their wishes. Those interested should state name, address, whether it is desired to take a degree in physics or in engineering, the name of the nearest technical college at which it is desired to attend and whether all-day Saturday or part of Saturday and evening work is preferred. In order to enable such courses to commence in September of this year immediate action is required and, therefore, an early reply would be appreciated.

The following two types of courses are proposed and the replies should state which is preferred

(1) A two-year course, leading to final B.Sc. or B.Sc.(Eng.) for engineers who have already passed Inter. or who will be able to pass Inter. this year.

(2) A similar course, starting in September and lasting, say, three years, for those who would be able to pass Inter. next year.

It will be appreciated that the Ministry of Education, to whom a copy of this letter is being sent, cannot be expected to provide such courses unless a genuine need for them exists.

Langley Park, Nr. Slough, Bucks. O. S. PUCKLE. R.F. Equipment, Ltd.

Transient Response of Filters

To the Editor, "Wireless Engineer"

SIR,—In reply to the letters in the March issue, I should like to make the following points. To take Dr. Belevitch's example, consider the circuit consisting of $2R$ and L in series. The symbolic equation $2R + pL = 0$ has the root $p_1 = -2R/L$ corresponding to the transient $\exp(p_1 t)$. Now to form the band-pass analogue, add $C = 1/\omega_0^2 L$ in series, where ω_0 is the natural frequency; i.e.,

$$\omega_0^2 = \frac{1}{LC} - \frac{1}{4}\left(\frac{2R}{L}\right)^2$$

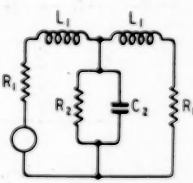


Fig. 1.

The symbolic equation becomes $p^2 - p_1 p + \omega_0^2 + \frac{p_1^2}{4} = 0$, yielding the roots $\frac{1}{2}\left(p_1 \pm 2j\omega_0\right)$, the corresponding transient $\exp(p_1 t/2)\cos\omega_0 t$ having the same envelope as the low-pass transient, except for the factor $\frac{1}{2}$ which will be discussed later.

Consider now the circuit consisting of $2R$ and C in parallel. The symbolic equation $1 + 2pRC = 0$ has the root $p_2 = 1/2CR$ corresponding to the transient exp. $(p_2 t)$. To form the band-pass analogue, add $L = 1/\omega_0^2 C$ in parallel with C, where ω_0 is given by

$$\omega_0^2 = \frac{1}{LC} - \frac{1}{4}\left(\frac{1}{2RC}\right)^2$$

The symbolic equation is then $p^2 - p_2 p + \omega_0^2 + \frac{p_2^2}{4} = 0$ yielding the roots $\frac{1}{2}\left(p_2 \pm 2j\omega_0\right)$, the corresponding transient again having the same envelope as the low-pass transient (apart from the factor $\frac{1}{2}$).

If two low-pass systems as described above are placed in series to form a low-pass system, it is easily verified that the band-pass analogue has a transient response envelope identical with the low-pass transient, provided

(a) we take ω_0 as a natural frequency

(b) the damping of each system is the same.

For all systems included in the above description it is then quite correct to say that " if the wave $\cos \omega_0 t \cdot \mathbf{1}$ is put into a band-pass network, the output wave is exactly the same as the output wave obtained when unit step is placed into the low-pass analogue."

However, if we attempt to validate this statement for more complex networks it is found to be untrue except in the case where the band-width is small compared with the mid-band frequency. Consider, for example, the network shown in the diagram Fig. 1.

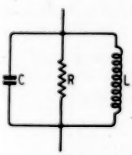


Fig. 2 (*above*).

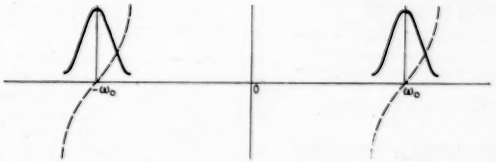
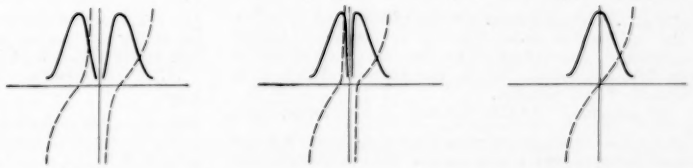


Fig. 3 (*right*).



If we have equal damping, i.e. $\alpha = R_1/L_1 = 1/R_2C_2$ we obtain as a symbolic equation of the low-pass network $(p + \alpha)^2 + \omega_s^2 = 0$ where $\omega_s^2 = 1/L_1C_2$. The roots of this equation correspond to the transient exp $(-\alpha t) \cos \omega_s t$.

The band-pass analogue, when the network is transformed as in the simple cases above, yields the symbolic equation,

$$\left[\left(p + \frac{\alpha}{2}\right)^2 + \omega_0^2\right]^2 + p^2\omega_s^2 = 0$$

where

$$\omega_0^2 = \frac{1}{L_1C_1} - \frac{\alpha^2}{4} = \frac{1}{L_2C_2} - \frac{\alpha^2}{4}$$

Now the symbolic equation *does not* correspond to the transient exp $(-\alpha t/2) \cos \omega_s t \cos \omega_0 t$ or to any such simple form as this. This may be seen to be so, since if we attempt to replace $p + \frac{\alpha}{2}$ by p (corresponding to removing exp $(-\alpha t/2)$), $\alpha/2$ does not disappear. The decay factor present in the symbolic equation, is indeed no longer equal to $-\alpha/2$.

In general, therefore, Belevitch and Thomson rightly state that when the damping is such as to make the natural frequency differ greatly from the resonant frequency defined by an LC product, the statement above in inverted commas is not true. This statement only holds when the Q's of the network are high. This corresponds to the requirement that the bandwidth must be small compared with the mid-band frequency.

Assuming the bandwidth is small compared with the mid-band frequency, the fact that we have not only to replace ω by $\omega - \frac{\omega_0^2}{\omega}$ but also double the bandwidth to obtain the band-pass analogue, may be explained by reference to the amplitude- and phase-frequency characteristics. Consider, for example, the characteristics of a simple tuned circuit, as shown in Fig. 2.

Now, if we increase the value of the inductance L, the value of ω decreases and as the response curves approach zero frequency their shapes begin to alter, Fig. 3(a). As L becomes very large the responses take the form (b).

It will be noted that the amplitude response at $\omega = 0$ is still zero, since we have an inductance whose impedance at zero frequency is zero. Now if we remove L completely the circuit has a zero-frequency response and the final response curves for the low-pass network have the form of Fig. 3(c).

It will be seen that for the band-pass case we have twice the bandwidth associated with the low-pass case, thus accounting for the fact that the time subsidence of transients for a band-pass network having a bandwidth small compared with the mid-band frequency, is twice as great as in the low-pass analogue. My final equation in the January issue referred to the band-pass case since I was making a comparison with C. C. Eaglesfield's result. There was, though, a certain amount of ambiguity.

It will be further noted that when the bandwidth considered is small compared with mid-band frequency, then the shapes of the band-pass characteristics tend to those of the low-pass characteristics. This follows putting $\omega = \omega_0 + \frac{\Delta\omega}{2}$ where $\Delta\omega/\omega_0 \ll 1$ since we have

$$\omega - \frac{\omega_0^2}{\omega} \approx \Delta\omega$$

The expression for the band-pass response in terms of $\Delta\omega$, corresponds to the expression for the low-pass response in terms of ω.

London, E.14. EDWARD T. EMMS.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

579 256.—Tone-control circuit comprising positive and negative feedback paths.

Standard Telephones and Cables Ltd. (communicated by International Standard Electric Corporation). Application date 25th April, 1944.

579 407.—Two-stage a.f. amplifier comprizing tuned circuits with positive reaction for emphasizing selected parts of the audio-frequency range.

B.P.L. (Instruments) Ltd. and B. Digby. Application date 25th April, 1944.

579 685.—Multistage amplifier for low frequencies or direct current, comprising valves with their anode-cathode paths in series across the h.t. supply, and valves with their cathodes in common.

H. L. Mansford. Application date 24th April, 1944.

AERIALS AND AERIAL SYSTEMS

579 655.—Transmitting and receiving aerials arranged in close proximity, but so as to allow distant signals to be received on or near the same frequency as the outgoing signals.

J. Robinson. Application date 1st June, 1943.

DIRECTIONAL AND NAVIGATIONAL SYSTEMS

578 479.—Navigational system of the kind in which the course is indicated by a constant phase-difference, characterized by the use of a method of modulation which keeps the course secret from unauthorized persons.

H. Fletcher. Application date 9th September, 1942.

578 550.—Radiolocation equipment, say for an aeroplane, in which the scanned field extends to the right and left of the direction of flight, both in front and to the rear of the moving craft.

Standard Telephones and Cables Ltd. (assignees of H. G. Busignies). Convention date (U.S.A.) 24th February, 1941.

578 690.—Pulse-generator of the multivibrator type, as used for radiolocation, in which the frequency of operation is stabilized by secondary impulses derived from the main pulses through delay networks.

Standard Telephones and Cables Ltd., B. B. Jacobsen and M. M. Levy). Application date 8th July, 1941.

578 737.—Quarter-wave frictionless coupling between the fixed and rotary sections of a coaxial line for feeding the rotating aerial, say, of a radiolocation set.

The General Electric Co. Ltd. and D. C. Espley. Application date 14th June, 1940.

578 773.—Eliminating undesired secondary lobes from a duplex aerial-system for marking-out a blind-landing glide-path.

Standard Telephones and Cables Ltd. and L. J. Heaton-Armstrong. Application date 30th October, 1942.

578 875.—Capacitance network for shaping the modulating voltages for radiolocation or like pulsed signals.

The British Thomson-Houston Co. Ltd. and K. J. R. Wilkinson. Application date, 8th December, 1941.

578 911.—Frictionless coupling between the fixed and rotary stubbed sections of a coaxial line for energizing a rotating aerial as used, say, for radiolocation.

The General Electric Co. Ltd. and D. C. Espley. Application date 26th August, 1941.

579 042.—Balanced-bridge coupling for allowing the same aerial-system to be used simultaneously with two direction-finders on the same or different wave-lengths.

Standard Telephones and Cables Ltd., C. W. Earp, I. R. J. James and R. F. Cleaver. Application date 9th February, 1940.

579 090.—Course-indicator of the overlapping-beam type in which the normal centre-zero meter is also adapted to give a positive indication of a failure in the transmission of signals.

Standard Telephones and Cables Ltd. and H. P. Williams. Application date 11th May, 1944.

579 154.—Radiolocation system in which successive exploring pulses alternate in frequency above and below a standard frequency.

A. D. Blumlein. Application date 29th March, 1940.

579 167.—Blind-landing system utilizing three overlapping and differently-modulated beams of infra-red rays.

E. T. J. Tapp and Vacuum Science Products Ltd. Application date 15th December, 1942.

579 170.—Discriminating device for automatically maintaining a constant phase and amplitude relation between the overlapping beams of a radio-navigational system.

W. J. O'Brien. Convention date (U.S.A.) 13th April, 1942.

579 274.—Amplifier with negative feedback for maintaining linearity in a remote-control goniometer device, such as the "mag-slip resolver" used in certain forms of radiolocation equipment.

A. C. Cossor Ltd. and L. H. Bedford. Application date 7th October, 1942.

579 346.—Radio-navigational system in which a mobile receiver ascertains the phase-difference between signals transmitted from spaced stations on a single frequency but with distinctive characteristics.

H. T. Mitchell and T. Kilvington. Application date 1st March, 1944.

579 360.—Directional system in which the azimuth

and elevation of a source of radiation are indicated by two separate traces on a cathode-ray tube.

H. A. M. Clark. Application date 18th November, 1938.

579 072.—Bridge-rectifier circuit for converting the dot and dash signals of a radio-approach system into equivalent signals of constant sign, for feeding a d.c. meter or indicator.

Standard Telephones and Cables Ltd. and J. D. Weston. Application date 11th May, 1942.

579 725.—Pulse-generators, for use in radiolocation, wherein intense modulating-voltages of short duration are built-up in the anode circuits of one or more valves when these are periodically biased to cut-off.

A. D. Blumlein and E. L. C. White. Application date 27th January, 1940.

RECEIVING CIRCUITS AND APPARATUS

(See also under Television)

578 512.—Selective receivers, particularly for use on mobile vehicles, in which modulating or mixer stages are combined with passive filter circuits.

The General Electric Co. Ltd., H. C. Turner and E. A. Fielding. Application date 12th November, 1941.

578 716.—Receiver for discriminating between two frequency-modulated signals by combining them with a third oscillation in a limiter valve, and selecting an image frequency from the output.

A. C. Cossor Ltd. and D. A. Bell. Application date 24th July, 1944.

578. 823—Receiver coupled to two alternative sources of supply through a rectifier circuit which automatically brings the second source into operation should the first one fail or be disconnected.

E. K. Cole Ltd. and L. W. D. Sharp. Application date 17th June, 1944.

578 876.—Mixing circuit comprising a cavity resonator with tuning probes and input and output electrodes for heterodyning ultra-short waves.

The British Thomson-Houston Co. Ltd. and T. H. Kinman. Application date 1st January, 1942.

578 880.—Band-pass filter circuit of the double-tuned type including means for adjusting the spacing and heights of the two resonance peaks.

Philco Radio and Television Corp. (assignees of W. E. Bradley). Convention date (U.S.A.) 24th July, 1942.

578 952.—The use of boron carbide in contact with a metal for mixing or rectifying centimetre waves.

The General Electric Co. Ltd. and J. W. Ryde. Application date 23rd November, 1940.

578 960.—Tracking circuit designed to maintain a constant tuning-relation with the main circuit over a wide range of operation.

The General Electric Co. Ltd. and A. Bloch. Application date 23rd October, 1942.

578 969.—Tunable coupling-network, particularly for suppressing the image-frequency in a super-heterodyne receiver.

Marconi's W.T. Co. Ltd. (assignees of W. F. Sands). Convention date (U.S.A.) 30th December, 1942.

579 012.—Compact tubular trimming unit, comprising an inductance and a capacitor, both separately adjustable.

Marconi's W.T. Co. Ltd. and E. R. Burroughes. Application date 26th April, 1944.

579 117.—Receiver in which the signals are divided between two paths of different electrical lengths, feeding a differential detector, in order to increase the signal-to-noise ratio.

Standard Telephones and Cables Ltd. and C. W. Earp. Application date 19th December 1941.

579 230.—Short-wave superheterodyne receiver in which a concentric-line resonator is arranged to stabilize the operation of the local oscillator.

Marconi's W. T. Co., Ltd. (assignees of K. G. MacLean). Convention date (U.S.A.) 30th July 1938.

579 300.—Chassis arrangement, and supporting brackets and pins, for mounting transportable radio-sets.

G. A. Laughton and H. N. Cox. Application date 17th April, 1944.

579 507.—Electronic-switching arrangement for coupling the spaced aerials, in a diversity or anti-fading system, to the common receiver.

Marconi's W. T. Co., Ltd. (assignees of M. G. Crosby). Convention date (U.S.A.) 6th May, 1943.

579 671.—Method of demodulating f.m. signals which includes the step of " clipping " the waves to approximately rectangular form, in order to offset the effect of accidental amplitude modulation.

Standard Telephones and Cables Ltd. (assignees of L. A. de Rosa). Convention date (U.S.A.) 13th November, 1942.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

579 031.—Mobile television equipment in which the orientation of a picture received, say from an aeroplane, is identified by a transmitted compass marking.

C. O. Browne and H. W. Hobbs. (Secret patent). Application dates 16th March and 20th April, 1938, *Published 24th May,* 1946.

579 273.—Television transmitters of the kind in which the scanning beam reaches the mosaic with substantially zero velocity, characterized by means for preventing so-called " crawling " distortion.

Marconi's W. T. Co., Ltd. (assignees of L. E. Flory, E. A. Massa, and G. A. Morton). Convention date (U.S.A.) 21st June, 1941.

579 482.—Method of inserting a comparatively-large viewing-screen into the spherical glass bulb of a cathode-ray television receiver.

J. L. Baird. Application date 28th April, 1944.

579 600.—Television transmitter in which undesired " shading " effects are reduced by a method which involves a periodic reduction or interruption of the scanning beam.

Hazeltine Corpn., (assignees of A. V. Loughren) Convention date (U.S.A.) 30th March 1943.

TRANSMITTING CIRCUITS AND APPARATUS

(See also under Television)

578 466.—Waveguide provided with internal plates or fins which serve to reduce the normal cut-off frequency of transmission.

The British Thomson-Houston Co. Ltd. Convention date (U.S.A.) 16th August, 1943.

578 597.—Waveguide fitted with internal apertured portions, forming variable reactors, for tuning or filtering purposes.

Western Electric Co. Inc. Convention date (U.S.A.) 30th July, 1942.

578 617.—Waveguide in which a transverse partition is suitably apertured to provide local resonance for energy of a given wavelength (divided out of 578 597.)

Western Electric Co. Inc. Convention date (U.S.A.) 30th July, 1942.

578 664.—Short-wave transmitter with a spark-gap system comprising two sections, one for rapidly establishing the discharge, the other for rapidly restoring the dielectric medium.

Standard Telephones and Cables Ltd. (communicated by International Standard Electric Corporation). Application date 15th February, 1944.

579 128.—Construction of balanced impedance couplings or connectors for coaxial feed-lines.

E. C. Cork. Application date 15th January, 1941.

579 567.—Heterodyne method of frequency control and stabilization for a combined transmitter and receiver set, working on frequency-modulated signals.

Marconi's W. T. Co., Ltd. and H. R. Cantelo. Application date 7th October, 1943.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

578 514.—Arrangement of the charging inductance in a triggered circuit for generating pulses of variable recurrence-frequency, or for developing a time-base for a c.r. tube.

M. E. Haine and Metropolitan-Vickers Electrical Co. Ltd. Approximate date 11th July, 1942.

578 732.—Generating pulses to convey speech or like signals by sweeping an electron beam repeatedly but variably across a target electrode.

Standard Telephones and Cables Ltd., and W. A. Beatty. Approximate date 21st March, 1940.

579 126.—Signalling system in which time-modulated pulses are used to control the initial amplitude of a series of damped oscillations, say for multiplex or secret working.

Standard Telephones and Cables, Ltd., and W. A. Beatty. Application date 10th November, 1939.

579 565.—Demodulating signals sent in the form of pulses, having a variable frequency or phase of repetition, by a method which includes the step of mixing them with saw-toothed waves of constant frequency.

A. C. Cossor, Ltd., and D. A. Bell. Application date 17th September, 1943.

579 652.—Time-base circuit and cathode-ray indicator for monitoring the width, phase, or amplitude modulation in a pulsed-signalling system.

C. J. Carter and Pye Ltd. Application date 18th December, 1942.

579 672.—Pulse-generating and modulating circuit, particularly suitable for multi-channel signalling systems.

Standard Telephones and Cables Ltd., P. K. Chatterjea and C. T. Scully. Application date 16th November, 1943.

579 673.—Pulse-generator with anode excitation through a spark-gap device, suitable for timed-pulse signalling, or for radiolocation.

Standard Telephones and Cables Ltd. (assignees of R. B. Hoffman). Convention date (U.S.A.) 23rd November, 1942.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

578 762.—Electron discharge tube in which quarter-wave "earthing" leads are incorporated in order to dissipate undesired potentials set up at certain points of the electrode-structure.

L. F. Broadway and E. C. Cork. Application date 6th August, 1940.

578 840.—Hollow resonator, as used for velocity-modulation, with a screw-controlled device which varies the tuning by deformation.

H. E. Holman and N. C. Barford. Application date 15th April, 1942.

578 848.—Electron discharge device for velocity-modulation in which lead-in conductors of the coaxial-line type are sealed into the glass tube.

The British Thomson-Houston Co. Ltd., and W. J. Scott. Application date 2nd March, 1943.

579 062.—Ultra-short-wave oscillator in which the electrode system is combined with two coaxial resonators, the vacuum tube forming one mechanical unit with the oscillation circuit.

Standard Telephones and Cables Ltd. (assignees of W. Hotine). Convention date (U.S.A.) 8th October, 1942.

SUBSIDIARY APPARATUS AND MATERIALS

578 730.—Circuit for giving a direct indication on a c.r. tube of the direction of a mobile craft or other body radiating infra-red rays.

H. A. M. Clark. Application date 23rd December, 1938.

578 818.—Frequency-measuring indicator of the kind in which each separate digit of the measured value is separately ascertained by heterodyning against certain standard frequencies.

The General Electric Co. Ltd., and F. C. F. Phillips. Application date 1st May, 1944.

578 844.—System for navigating ships by utilizing the lines of force produced by feeding direct, alternating or pulsating currents into the sea at two or more spaced points.

W. Stern. Application date 3rd November, 1942.

578 877.—Hollow resonator with symmetrically-arranged movable plugs or plungers for tuning control.

G. B. Banks and N. Levin. Application date 8th May, 1942.

578 901.—Tuned circuit for generating and controlling the output of large current-pulses, such as are used for welding operations.

The General Electric Co. Ltd., and E. Friedlander. Application date 12th July, 1944.

578 923.—Portable thermo-electric generator, suitable for charging accumulators.

Eaton Manufacturing Co. Convention date (U.S.A.) 22nd April, 1943.

578 955.—Arrangement and dimensions of thin wires for supporting a piezo-electric oscillator.

The General Electric Co. Ltd., and R. S. Rivlin. Application date 29th October, 1941.